Earth to Mars areostationary mission optimization analysis

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Abstract

Mars has become one of the priorities in the planetary exploration programs. The analysis of space mission costs has become a key factor in mission planning. The determination of optimal trajectories aiming to lower costs in terms of impulses allows for more massive payloads to be transported at a minimum energy cost. Areostationary satellites are considered the most efficient and robust candidates to satisfy the control needs of the missions to Mars. Mars Areostationary Relay Satellites providing continuous coverage of a specific region of Mars are being considered for a near future.

In this work, we analyze the optimization of an areostationary mission. We first determine the launch and arrival dates for an optimal minimum energy Earth-Mars transfer trajectory. Then, the minimum thrust maneuvers to capture the spacecraft from the hyperbolic arrival trajectory to Mars and place it in the areostationary orbit are analyzed.

Context of the research: Previous studies

The number of missions to Mars has increased over the last years, particularly robotic missions which need to be tele-commanded from the Earth. The need to control the different missions in Mars in almost real time with a relay system that provides continuous coverage of a specific region has been proposed by several authors such as Edwards et al. (2001)(1) , Edwards et al. (2007)(2), Haque (2011)(3) , Podnar et al. (2010)(4) and Montabone (2020) (5).

Satellites in areostationary orbit are considered efficient candidates to satisfy telecommunication needs and supervision of tele-robotic exploration in foreseen missions to Mars. Areostationary satellites combine both the continuous coverage over the same Mars localization and the return of engineering and scientific instrument data in almost real time needed for the tele-robotic exploration in the coming set of Mars science missions. An areostationary satellite, in analogy with a geostationary satellite, would be in a circular areoequatorial orbit, i.e. with null inclination and eccentricity values, with a semi-major axis of 20428 km to be at rest with respect to the rotating Mars with a period of 1 Martian sidereal day (sol) of 88775.244 s.

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(1) C. Edwards, J. Adams, D. Bell, R. Cesarone, R. Depaula, J. Durning, T. Ely, R. Leung, C. McGraw, S. Rosell, Strategies for telecommunications and navigation in support of Mars exploration, Acta Astronaut. 48, 661–668, 2001.
(2) C. Edwards, R. DePaula, Key telecommunications technologies for increasing data return for future Mars exploration, Acta Astronaut. 61, 131–138, 2007.
(3) S. Haque, Broadband communication between Earth and Mars using linear circular commutating chain, in: Integrated Communications, Navigation and Surveilance Conference (ICNS), 2011.
(4) G. Podnar, J. Dolan, A. Elfes, Telesupervised robotic systems and the human exploration of Mars, J. Cosmol. 12, 4058–4067, 2010.
(5) Montabone, L. et al, Observing Mars from Areostationary Orbit: Benefits and Applications, MEPAG, 2020.

Work methodology: Trajectory determination and optimization

1. Elliptic heliocentric orbit optimization

Genetic algorithms are used to obtain the minimum energy Earth-Mars transfer trajectory according to the cost function:

$$C = W_{C_3} C_3 + W_{V_\infty} V_\infty,$$

being C_3 the characteristic energy at the launch from Earth, V_∞ the areocentric ecliptic velocity at the sphere of influence of Mars, and W_{C_3} and W_{V_∞} the associated weights of C_3 and V_∞, respectively. The minimum of function C provides optimal launch and arrival dates for the mission.

Determining the optimal elliptic orbital transfer parameters by minimizing C by exhaustive search or analytical methods is infeasible as for each Earth departure date and Mars arrival date combination the Lambert's problem needs to be solved:

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3},$$

$$\vec{r}(t_1) = \vec{r}_E,$$

$$\vec{r}(t_2) = \vec{r}_M + \vec{r}_{s_M}.$$

Genetic algorithms are heuristic algorithms that explore different solutions amongst a defined population, finding a possible minimum solution. Lambert's problem shall be solved for each member of the population following the procedure of Gooding (1988)(6), having the orbital elements of the spacecraft's orbit once the solution is achieved.

2. Elliptic heliocentric and hyperbolic areocentric orbits matching

Hyperbolic orbit parameters are obtained by fixing inclination and periapsis radius desired for the final orbit using the following iterative procedure to match the heliocentric and hyperbolic orbits:

$$\vec{r}_{s_M}^{(i+1)} = \boldsymbol{g}\left(\boldsymbol{f}\left(\vec{r}_{s_M}^{(i)}\right)\right), \qquad \vec{V}_{s_M}^{(i+1)} = \boldsymbol{h}\left(\boldsymbol{f}\left(\vec{r}_{s_M}^{(i)}\right)\right)$$

The algorithm converges when $\left|\boldsymbol{h}\left(\boldsymbol{f}\left(\vec{r}_{s_M}^{(i)}\right)\right) - \boldsymbol{f}\left(\vec{r}_{s_M}^{(i)}\right)\right| < \varepsilon$ or equivalently when $\left|\vec{V}_{s_M}^{(i+1)} - \vec{V}_{s_M}^{(i)}\right| < \varepsilon$, where $\boldsymbol{f}\left(\vec{r}_{s_M}^{(i)}\right) = \vec{V}_{\infty_M}^{(i)}$, is the Lambert's problem solution, and $\boldsymbol{g}\left(\vec{V}_{\infty_M}^{(i)}\right) = \vec{r}_{s_M}^{(i+1)}$ and $\boldsymbol{h}\left(\vec{V}_{\infty_M}^{(i)}\right) = \vec{V}_{\infty_M}^{(i+1)}$ are the functions given by Battin (1999)(7) to compute the position and velocity in the hyperbolic areocentric orbit at the sphere of influence of Mars.

3. Manoeuvres optimization at Mars sphere of influence

Now we search a solution that minimizes the total impulse cost, ΔV_M:

$$\Delta V_M = \Delta V_c + \Delta V_i + \Delta V_A + \Delta V_P$$

where ΔV_c is the capture manoeuvre, ΔV_P and ΔV_A are the Hoffman transfer manoeuvres from the capture orbit to target the areostationary orbit; and ΔV_i is the inclination correction manoeuvre to reach the final zero desired inclination.

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(6) Gooding, R.H., On the solution of Lambert's orbital boundary-value problem, Royal Aerospace Establishment, 1988

(7) Battin, R.H., An Introduction to the Mathematics and Methods of Astrodynamics, AAIA Educational Series, 1999

Simulation results (I)

Elliptic heliocentric orbit optimization

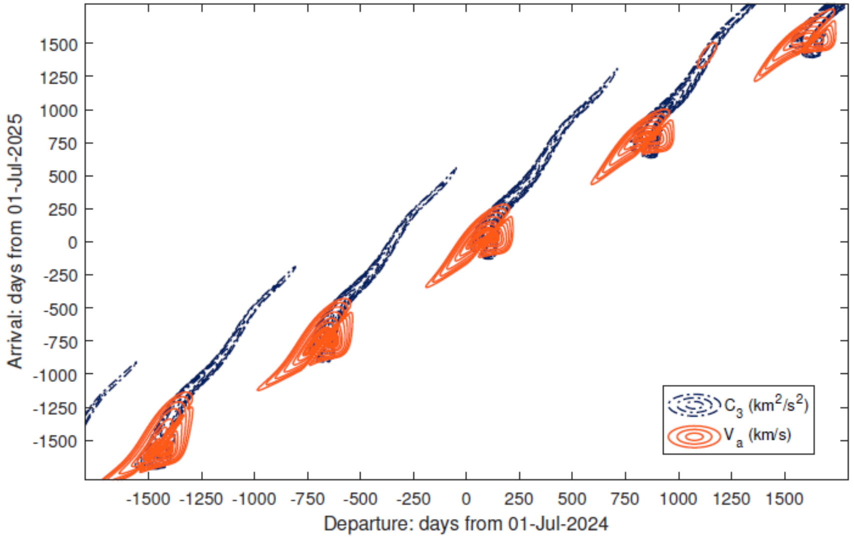

***Figure 1**: Porkchop plots for arrival velocity V_∞ and characteristic energy at departure C_3 over a period of 10 years.*

Plots show that minimums are achieved every 780 days approximatelly, that coincide with the sinodic period of Mars. The launch window of 2020 has been chosen for optimization.

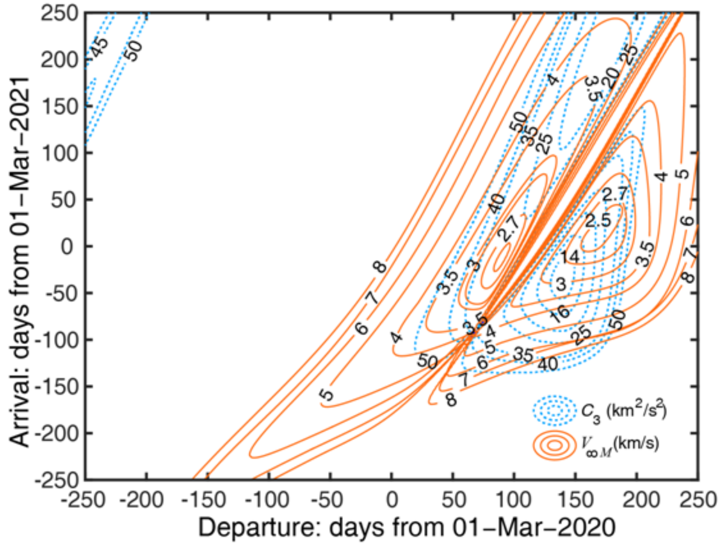

***Figure 2**: Porkchop plots for departure dates on March 2020 and arrival on March 2021.*

The figure shows that minimum transfer is achieved around July 2020 for departure and January 2021 for arrival.

Genetic algorithms are now used to narrow the optimal dates.

Pop.	Crossover	Selection	C.P.U. Time (s)	Departure time	TOF (days)	C_3 (km^2/s^2)	$V_{\infty M}$ (km/s)
Equal weight factors of $W_{C_3} = 1$ and $W_{\infty M} = 1$ Departure date 20-Jul-20							
100	Heuristic	remainder	10.86	01:19:08	196.9397	13.1268	2.7683
		stoch. unif.	10.36	01:05:27	196.9253	13.1264	2.7686
	Scattered	remainder	10.30	01:04:01	196.9501	13.1265	2.7686
		stoch. unif.	10.49	01:01:17	196.9281	13.1264	2.7687
	Single point	remainder	10.14	01:06:02	196.9657	13.1270	2.7681
		stoch. unif.	10.35	01:14:40	196.9566	13.1270	2.7681
500	Heuristic	remainder	48.30	01:06:27	196.9288	13.1264	2.7686
		stoch. unif.	47.86	01:13:05	196.9420	13.1267	2.7684
	Scattered	remainder	48.31	01:12:22	196.9420	13.1267	2.7684
		stoch. unif.	47.60	01:21:17	196.9537	13.1270	2.7681
	Single point	remainder	47.32	01:24:10	196.9407	13.1268	2.7683
		stoch. unif.	47.19	01:16:41	196.9450	13.1268	2.7683
2000	Heuristic	remainder	188.06	01:04:09	196.9393	13.1266	2.7685
		stoch. unif.	186.57	01:07:45	196.9332	13.1265	2.7686
	Scattered	remainder	222.98	01:16:41	196.9450	13.1268	2.7683
		stoch. unif.	229.03	01:10:38	196.9361	13.1266	2.7685
	Single point	remainder	238.38	01:08:54	196.9343	13.1266	2.7685
		stoch. unif.	237.49	01:26:54	196.9480	13.1270	2.7681

***Table 1**: Genetic algorithms performance depending on cross and selection functions, tolerance and population size. Genetic algorithms executed for 2020 launch window.*

Results show no differences in the resulting launch energy and arrival velocity depending on the selection and cross functions chosen. Performance is better when smaller population size is used.

Simulation results (II)

Elliptic heliocentric and hyperbolic areocentric orbits matching

Fixed initial conditions for the orbit matching are periapsis radius, fixed to the areostationary value $r_p = 20428\,km$, and hyperbolic inclination, which is fixed to different values to see the impact of this value in the final orbit and therefore in the manoeuvres. Final results show:

- Initial inclination and periapsis radius are obtained at the end of the iterative procedure,
- Velocity converges after three iterations at 10^{-4} km/s accuracy,
- Position converges after 8 iterations.

Table 2 shows the accuracy results for each iteration of the procedure. It is observed that once the algorithm converges, there is continuity at the sphere of influence boundary in position, and the velocity suffers a discontinuity of 6.02×10^{-4} km/s.

Iteration	$\left\|\vec{V}_{s_M}^{(i+1)} - \vec{V}_{s_M}^{(i)}\right\|$ (km/s)	$\left\|\vec{r}_{s_M}^{(i+1)} - \vec{r}_{s_M}^{(i)}\right\|$ (km)	Inclination (deg)
1	5.922467395733E-04	577239.19747	50
2	6.024297860559E-04	13144.87475	50
3	6.020044068105E-04	157.24817	50
4	6.020031157060E-04	2.27300	50
5	6.020030576542E-04	0.03603	50
6	6.020030569832E-04	0.00057	50
7	6.020030569710E-04	0.00001	50
8	6.020030569708E-04	0	50

Table 2: *Iterative procedure results for an inclination value of 50 degrees.*

Manoeuvres optimization at Mars sphere of influence

Key parameters for manoeuvres optimization are inclination of the arrival orbit and periapsis radius of the hyperbolic orbit achieved. Manoeuvres results for a periapsis equal to the areostationary semimajor axis show that:

- The capture manoeuvre ΔV_c to avoid that the probe leaves its sphere of influence on a flyby trajectory is constant.
- The inclination manoeuvre ΔV_i to achieve the objective zero inclination is the main contribution to the total impulse needed.
- The Hohmann impulses $\Delta V_H = \Delta V_A + \Delta V_P$ are null for the example in Figure 3 as the areostationary objective radius has been fixed with the capture manoeuvre.

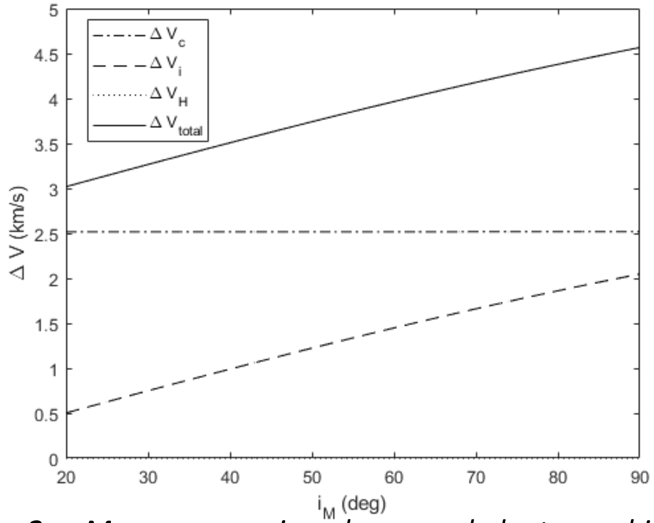

Figure 3: *Manoeuvres impulse needed to achieve the areostationary orbit.*

As conclusion, the minimum cost corresponds to a capture manoeuvre at a periapsis with value equal to the areostationary semi-major axis and choosing the minimum objective inclination that could be achieved, which depends on the declination of the incoming asymptote given by $\vec{V}_{\infty M}$

Summary

The determination of optimal transfer trajectories from Earth to Mars aiming to lower costs in terms of impulses has become a key factor in mission planning, allowing for more massive payloads to be transported by Solar Electric Propulsion at a minimum energy cost. In this work we first focus on the determination of optimal interplanetary trajectories from Earth to Mars by minimizing the total required impulses magnitudes in the major mission phases: Earth departure, interplanetary targeting orbit and Mars arrival. Such analysis is done by solving the Lambert orbital boundary value problem and investigating the optimal departure and arrival windows.

The results of the analysis lead to an optimal choice of departure and arrival dates that minimize the necessary amount of needed impulses for an Earth to Mars mission. Once solved the optimal hyperbolic arrival trajectory about Mars, optimal necessary maneuvers in order to capture the orbiter and to and place it the areostationary orbit are analyzed.

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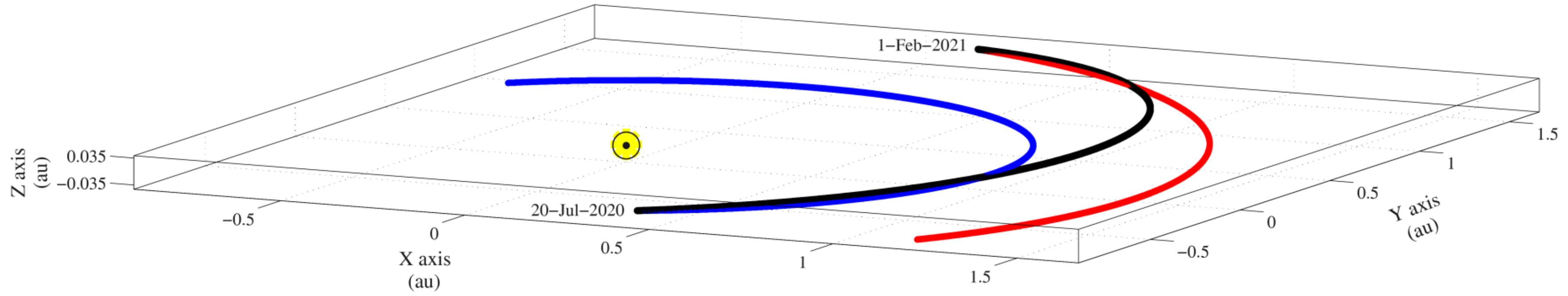

Figure 4: *Optimal transfer orbit from Earth to Mars from 20th July 2020 to 1st February 2021.*