COMPARISON OF DIFFERENT ADVECTION SCHEMES FOR 3D ATMOSPHERIC CODES

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ABSTRACT

We present a study of the effect of different advection schemes in a benchmark case for 3D atmospheric codes called Robert Rising Bubble. This case has been used for different authors to validate their models or numerical implementations due to the high sensitivity of the final state on the technique being used. The results obtained show that some total variation diminishing (TVD) schemes are not capable of reproducing correctly the morphology of the perturbation and that a Spectro-Consistent scheme is much more realistic, yet it needs a dissipation mechanism to overcome the spurious oscillations that appear. Moreover, we present additional statistics of the results and compare them to the predicted tendencies given by similarity theory.

CONTEXT OF THE RESEARCH

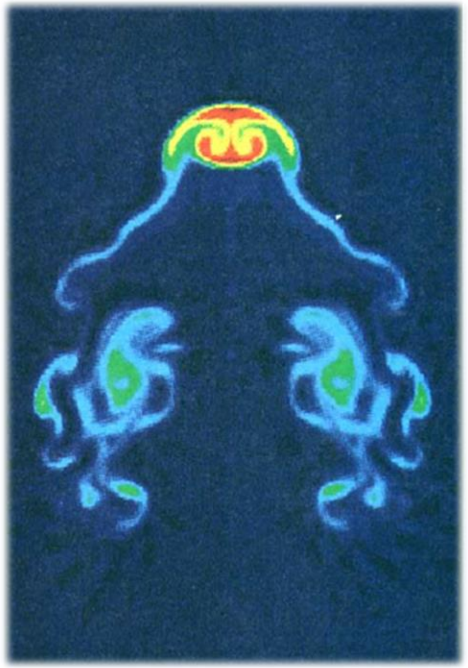
The beginning

A classic benchmark case for 3D atmospheric code is that of reproducing a thermal bubble rising due to buoyancy forces. This version of the experiment is carried out by introducing a uniform perturbation in the potential temperature (Carpenter, 1989).

A new change is introduced

In (Robert, 1993) a variation of this benchmark case is presented. It is a subtle change on the initial conditions of a thermal bubble making it vary in a Gaussian shape rather than a uniform perturbation. The results presented are only qualitative.

Note: *Some authors changed the function of the perturbation from a Gaussian to a cosine, yet it yields to very similar results.*



Original result. Extracted from (Robert, 1993)

Nowadays

Modern authors still use it for new atmospheric models with different numerical integration techniques. See for example (Li, 2019) and (Guerra, 2016) among others. Again, the results presented are only qualitative having no quantitative way to compare with.

Our contributions

We will reproduce the benchmark using a Boussinesq approximation using the following schemes:

- Spectro-Consistent
- MUSCL (TVD)
- VanLeer (TVD)
- Upwind

We will compare the results with the prediction of similarity theory and present additional quantitative measurements, based on the kinetic energy evolution and the maximum vertical velocity.

METHODOLOGY

Boussinesq approximation for stratified compressible flow as defined in (Durran, 2007):

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} - f(\vec{u} \times \vec{k}) = -c_p \theta_0 \nabla \pi' + \frac{\theta'}{\theta_0} g\vec{k} + \vec{F}$$

$$\nabla \cdot \vec{u} = 0$$

$$\frac{D}{Dt}\left(\frac{\theta'}{\theta_0} g\right) + N^2 w = 0$$

$$N^2 = \frac{g}{\theta_0}\frac{d\theta_0}{dz}$$

$$\pi = \left(\frac{p}{p_0}\right)^{\frac{R}{c_p}} = \pi_0 + \pi'$$ Exner function

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}} = \theta_0 + \theta'$$ Potential temperature

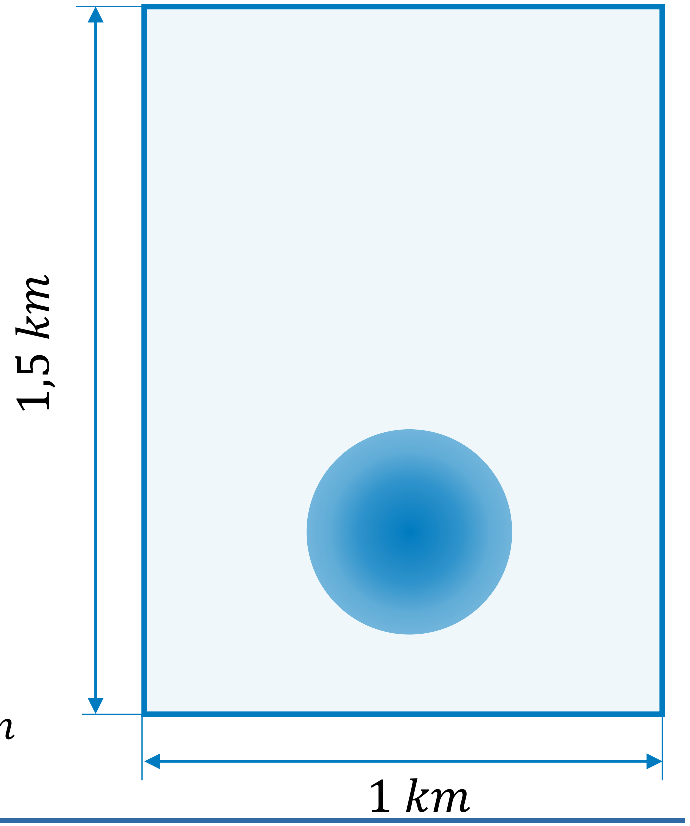
- Solved using a Fractional Step Method.
- Second order Adams-Bashforth for time integration with adaptive time step.
- Fourier solver for the Poisson equation.
- The code can run in parallel.
- 512×512 control volumes.

Initial conditions as stated in (Li, 2019) are:

$$\theta' = \begin{cases} 0 & if\ r > r_c \\ \frac{\theta_c}{2}\left[1 + \cos\left(\frac{\pi r}{r_c}\right)\right] & if\ r \le r_c \end{cases}$$

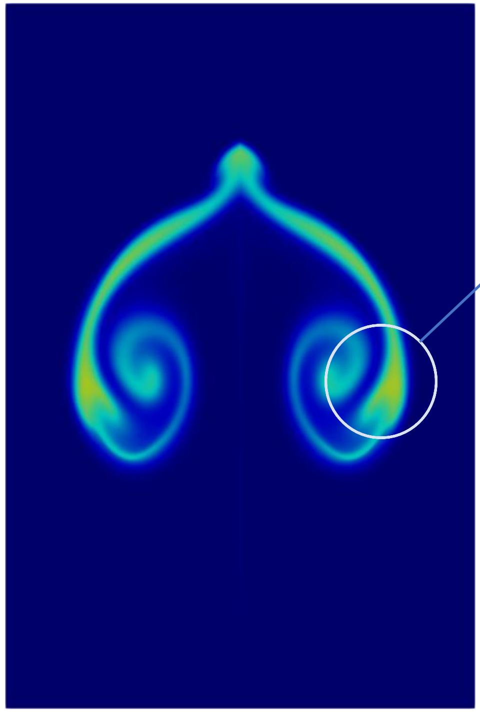
$$r = \sqrt{(x - x_c)^2 + (z - z_c)^2}$$

$t \in [0, 1200]\ s$ $\quad r_c = 250\ m$ $\quad \theta_0 = 303{,}15\ K$ $\quad f = 0$ $\quad \vec{F} = 0$ $\quad x_c = 500\ m$ $\quad z_c = 260\ m$



RESULTS

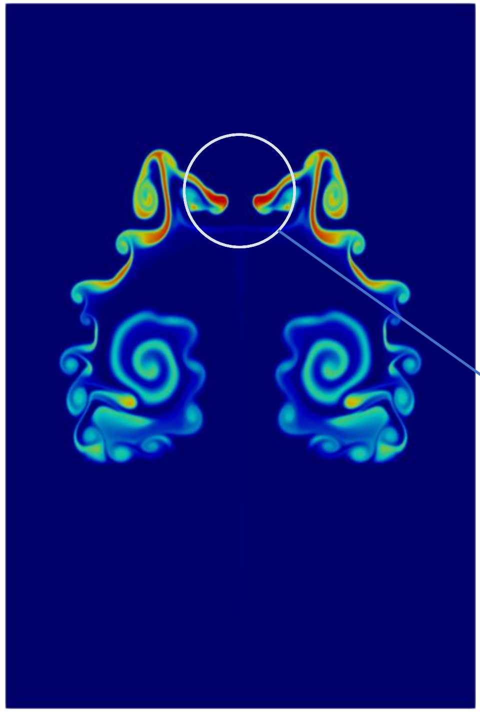
Snapshots of the perturbation of potential temperature at time $t = 16\,min$

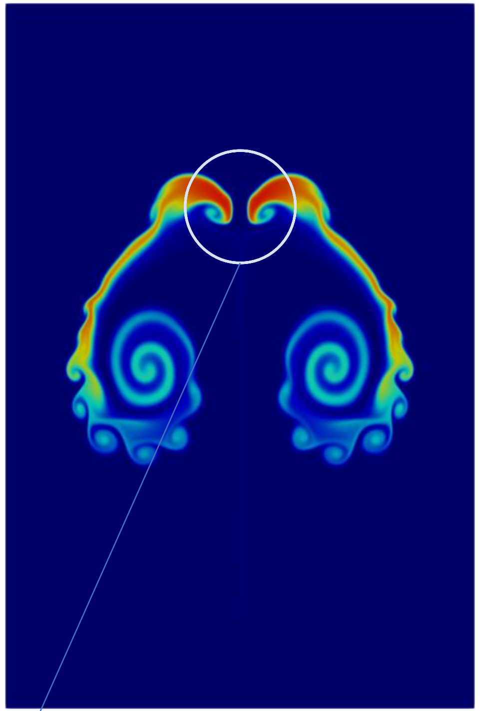


Upwind

No Kelvin-Helmholtz instability

MUSCL

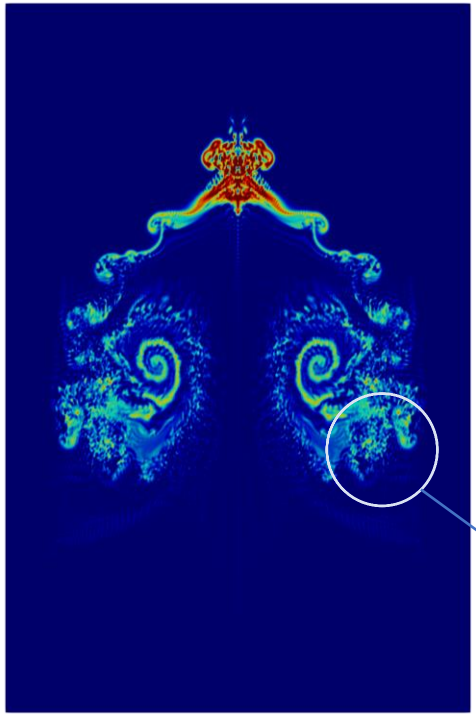




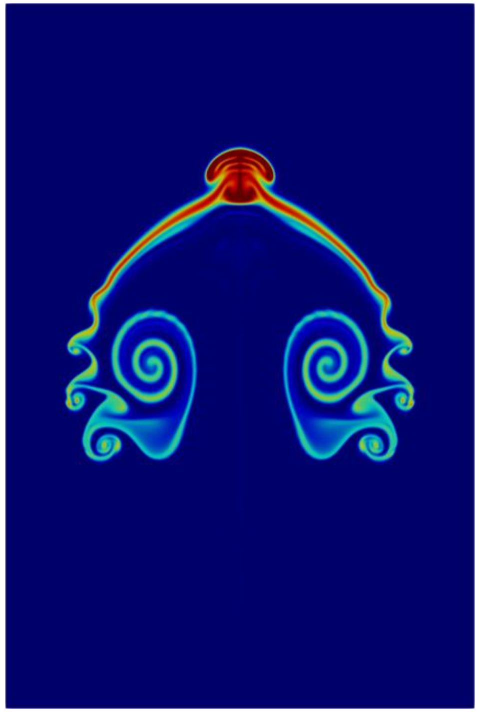
VanLeer

The bubble rips apart

Spectro-Consistent



High frequency noise



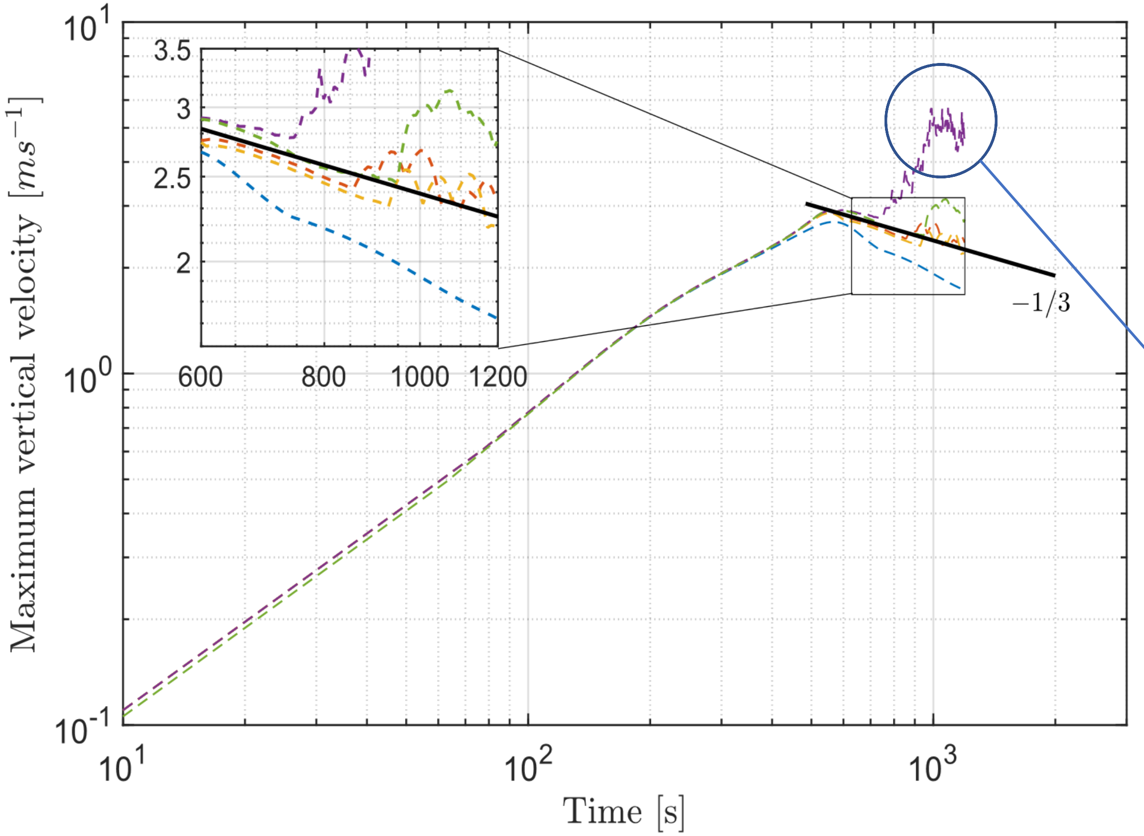
Spectro-Consistent with hyperviscosity filtering

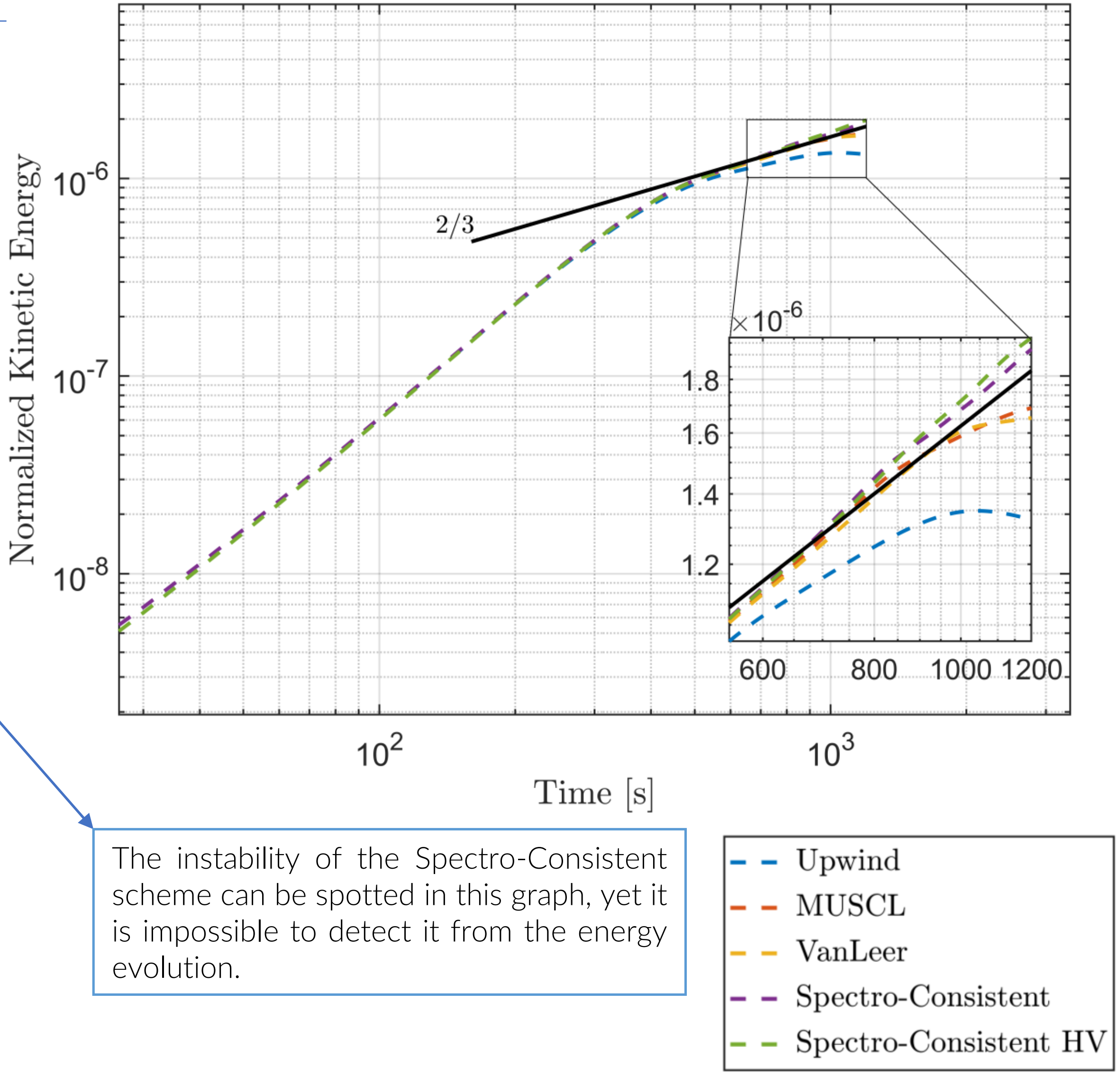
RESULTS

As stated in (Lilly, 1962) similarity theory predicts the following evolution for the developed flow:

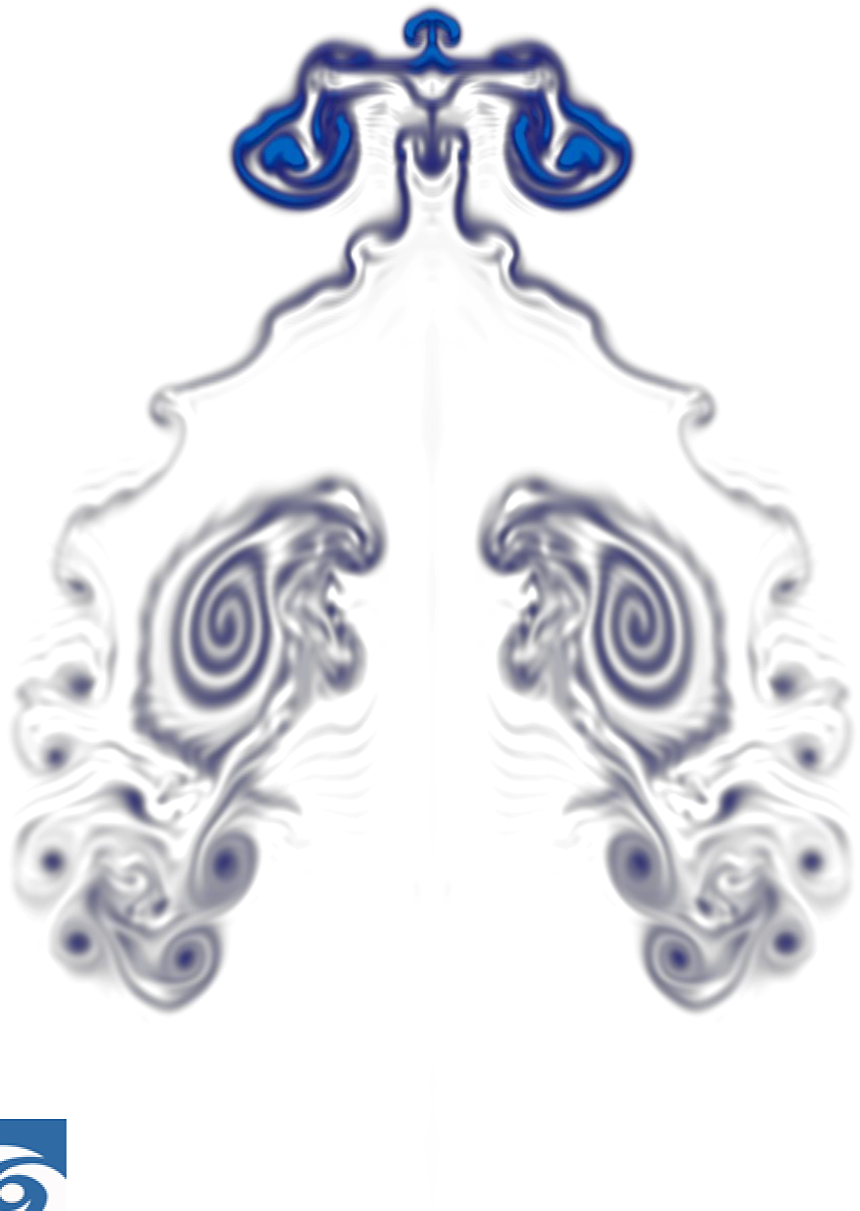
$$\iint E_k \, dxdz \propto t^{2/3}$$

$$w \propto t^{-1/3}$$





The instability of the Spectro-Consistent scheme can be spotted in this graph, yet it is impossible to detect it from the energy evolution.



IMPACT AND FUTURE WORK

With the results here presented we have shown that:

- The advection scheme must be chosen wisely if an atmospheric phenomena is to be reproduced since the upwards convective movement is very sensitive.
- TVD schemes are not capable of reproducing correctly this experiment, although the statistics presented demonstrate that they follow the theoretical trend.
- A Spectro-Consistent scheme alongside the hyperviscosity filter is capable of reproducing this experiment.

The future work might be centered in:

- Using this code with the Boussinesq approximation to reproduce planetary atmospheric phenomena (at the scales in which the model is suitable).
- Explore new ways to mitigate the spurious oscillations of the Spectro-Consistent scheme (e.g. filters based on entropy evolution).
- Implement a LES model for turbulence.
- Migrate to another approximation, such as the anelastic, with the knowledge of the effects of the different advection schemes.