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Fractal analysis of noise in pulsating stars

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Abstract

With the aim of characterizing the noise which is found in data series coming from missions like CoRoT or Kepler, a method for fractal noise analysis is presented. With this technique correlations or anticorrelations in a data series are brought into relief making possible to determine the scales involved. Further, the timescales found are shown to be a good signature of the physical nature of the noise.

1 Introduction

In the previous two decades a huge amount of data has become available on the stellar pulsations thanks to space missions like CoRoT[1], Kepler, Most and also Soho in the case of the Sun. These data allow us through their inversion to determinate the interior structure of the stars contrasting, in this way, the stellar evolution models. At the same time, with the unprecedented photometric accuracy of satellites like CoRoT, new perspectives have been opened and new problems have been found.

Since one year it has been found that some of the stars located in the instability strip of the delta Scuti in the HR-diagram like HD50844 [7], and HD174936 [1], appear to show a higher abundancy of oscillation frequencies than what theoretical models predict. More recently, it has been hypothesized in [5] that, in spite of the slim convective envelope, the granulation could have a non-negligible impact on the wealthy frequency spectra detected in these stars.

In this work, we try the fractal dimension of a light curve, determined through the Rescaled Range Analysis (R/S), as a tool to evaluate the impact of granulation on the frequency spectra of pulsating stars.

[1]The CoRoT space mission, launched on December 27, 2006, has been developed and operated by the CNES, with the contribution of Austria, Belgium, Brasil, ESA, Germany, and Spain.

2 Fractal analysis

Complex systems with turbulent flux characterized by a hierarchy of scales, as the case of the solar granulation, has been studied historically based on notions of fractal theory [2, 4]. In these works fractal dimension of solar granulation is determined. Fractal dimension tells us how a scale covers the space [6]. The idea is to verify the extent to which the cascade from larger to smaller scales follows a statistical self-similar behavior. This was suggested first by Kolmogorov in 1941 for the turbulent cascade in fluid turbulence. In the literature there are no studies about fractal dimension based on stellar light curves, that is the subject of this work.

In [2] analyzing 42742 solar granules by a spectrometer they found a fractal dimension of $d = 1.3$ for small scale granules, and $d = 2$ aprox. for the big ones, with a soft continuous transition between both regimes. In the same work it is shown that the fractal dimension for isobars in the case of homogeneous, isotropic turbulence is $d = 4/3$ and for isotherms $d = 5/3$, proving in this way, the relation between granulation and turbulence. Taking the hypothesis of spatial-temporal coherence of the granules as a starting point, in this work we make use of the data series from CoRoT and Soho (in the solar case), in order to find the fractal dimension of the granulation in different stars. The fractal dimension is linked with Hurst coefficient by this relation: $d = T + 1 - H$, where T is the topological dimension, then: $d = 2 - H$.

This parameter is a measure of the memory of the time series, that is:

- if $H = 0.5$ the series has no memory (random walk),
- if H is near 0.5 the series has short-range dependence (SRD),
- if $0.5 < H < 1$ the series has long-range dependence (LRD),
- if $0 < H < 0.5$ the series has LRD with an inverted trend.

Hurst parameter can be calculated with a low computational cost making use of a technique known as Rescaled Range Analysis (R/S). This technique is similar to the perimeter-area technique used by [2] but in this case we use timescales instead of perimeters, and variations of the time series instead of areas.

The R/S analysis consists in these steps:

1. Calculate the mean and substract it from the original series, this results in X_i.
2. A certain number of timescales is chosen: $t = 1, 10, 10^2, 10^3, ..,$ for example.
3. The cumulative deviate is found: $Y_t = \sum_{i=1}^{t} X_i$ for $t = 1, 10, \ldots, n$.
4. The range series is built from the differences of the maximum and the minimum of each subseries: $R_t = \max(Y_1, Y_{10}, \ldots, Y_t) - \min(Y_1, Y_{10}, \ldots, Y_t)$ for $t = 1, 10, \ldots, n$.
5. The standard deviation series σ_t is calculated from each subseries and its partial mean.

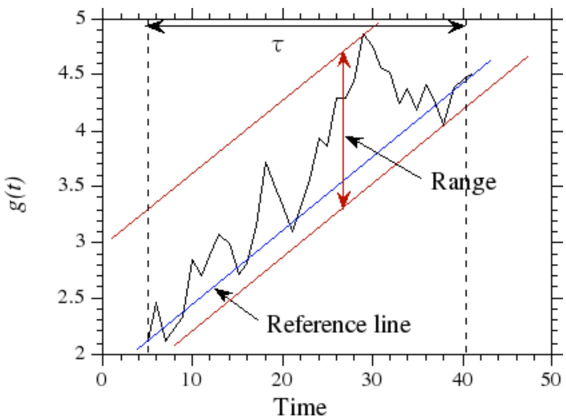

Figure 1: This shows how the Rescaled Range Analysis works. In the example instead of correcting the series by the mean, a reference line is chosen and the differences of the maximum and the minimum are calculated from it for each timescale.

6. Finally the reescaled range series is found: $(R/S)_t = \frac{R_t}{S_t}$ for $t = 1, 10, \ldots, n$.

There is a scheme of this technique in Fig. 1.

3 Results

Granulation is usually considered a noise component of the light curve but, contrary to what happens with white noise, which has a flat frequency spectrum, granulation is described by a power law [3]. In [5] this power law model is fitted to the frequency spectrum obtained from the light curves of the delta Scuti stars HD50844, and HD174936, proving that the abundance of frequencies in theses stars is reduced considerably when substracting the granulation model.

As fractals can be considered a form of colored noised, which by definition has a power law spectrum, we try here the R/S technique in order to find the Hurst exponent for these stars, and consequently the fractal dimension.

Figure 2 and Fig. 3 show the solar case and HD49933, which is one of the solar stars which has been studied profoundly and its granulation sign was discovered recently. A Hurst exponent near 0.5 is expected for these stars and others showing granulation because it is related to short-range dependence. In Table 1 the fractal dimension and Hurst exponent for the 4 cases studied are found. It is remarkable at most that only those cases whose granulation has been uncover clearly show a Hurst exponent near 0.5. In the two main cases of study, which are HD50844, and HD174936 the Hurst exponent is almost 1. That implies that there is no fractality because the fractal dimension coincides with the topological one.

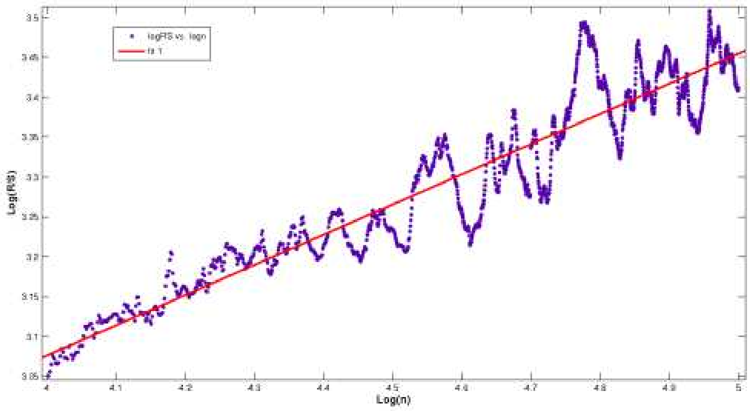

Figure 2: Rescaled Range Analysis of the Sun. The data comes from the VIRGO instrument onboad the SOHO satellite.

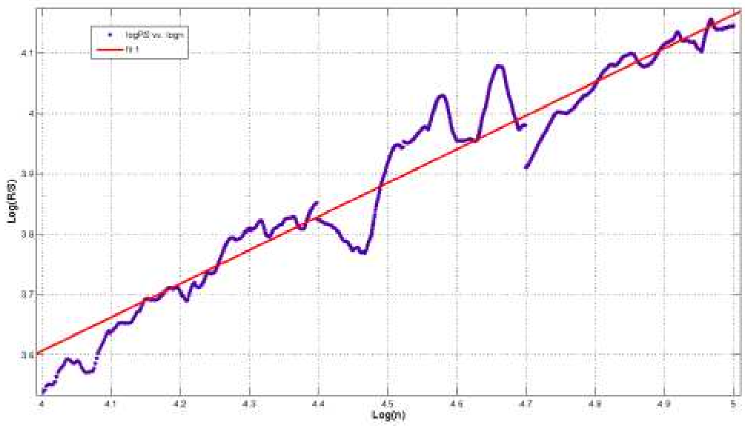

Figure 3: Rescaled Range Analysis of HD49933. The data comes from the Seismofield camera of CoRoT satellite.

Table 1: Hurst exponent and fractal dimension for the stars studied

Star Name	Hurst exponent	Fractal dimension
Sun	0.38	1.62
HD49933	0.56	1.44
HD50844	1.	1.
HD174936	1.	1.

4 Conclusions

The determination of the fractal dimension from light curves of pulsating stars seems to be tentatively a suitable tool for deciding whether a star have granulation or not. Our results agree with the expectations coming from the theory of the stellar interior.

The problem is that following this procedure we obtain only partial information about the statistical properties of the system. Furthermore, the range of scales under which the system presents scaling properties cannot be known a priori. Thus further tests are needed to prove the consistency of this method.

Acknowledgments

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