Highlights of Spanish Astrophysics VI, Proceedings of the IX Scientific Meeting of the Spanish Astronomical Society held on September 13 - 17, 2010, in Madrid, Spain. M. R. Zapatero Osorio et al. (eds.)

A multifrequency method based on the matched multifilter for the detection of point sources in CMB maps

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Abstract

In this work we deal with the problem of simultaneous multifrequency detection of extragalactic point sources in maps of the Cosmic Microwave Background. We developed a linear filtering technique that takes into account the spatial and the cross-power spectrum information at the same time.

1 Introduction

A big effort has been devoted to the problem of detecting point sources in Cosmic Microwave Background (CMB) experiments. The main reason is that the point sources contaminate the CMB radiation. It is therefore necessary to detect the maximum possible number of extragalactic point sources (EPS) and to estimate their flux with the lowest possible error. However, EPS are not just a contaminant that should be eliminated. They are a very important source of knowledge from the point of view of extragalactic astronomy (to derive source number counts and spectral indices, to constrain evolutive models, to study source variability, etc).

The detection and estimation of the flux of EPS are a difficult task. The main reason for this is that the many different types of EPS distributed in the sky form a very heterogeneous set of objects that do not have a common spectral behaviour in general. Our goal is to detect the highest number of EPS. For this reason, in order to reduce the threshold detection level of point sources, we use multi-wavelength information: statistical information of the background and the spatial profile of the sources for both channels at the same time. We also take into account the spectral behaviour of the sources without making any a priori assumption (for a more detailed description of the method, the simulations used and the obtained results, see [7]).

2 Method and simulations

In the multi-frequency approach we take into account the statistical correlation of the noise between different frequency channels and the frequency dependence of the sources. Now let us model the background $n_\nu(\mathbf{x})$ as a homogeneous and isotropic random field with average value equal to zero and crosspower spectrum $P_{\nu_1\nu_2}$ defined by:

$$\langle n_{\nu_1}(\mathbf{q}) n^*_{\nu_2}(\mathbf{q}')\rangle = P_{\nu_1\nu_2}\delta^2_D(\mathbf{q}-\mathbf{q}'), \tag{1}$$

where $n_\nu(\mathbf{q})$ is the Fourier transform of $n_\nu(\mathbf{x})$ and δ^2_D is the 2D Dirac distribution. Let us define a set of N linear filters ψ_ν that are applied to the data y_ν

$$w_\nu(\mathbf{b}) = \int d\mathbf{x}\; y_\nu(\mathbf{x})\psi_\nu(\mathbf{x};\mathbf{b}) = \int d\mathbf{q}\; e^{-i\mathbf{q}\cdot\mathbf{b}} y_\nu(\mathbf{q})\psi_\nu(q). \tag{2}$$

Here $\mathbf{b}$ defines a translation. The right part of Eq. 2 shows the filtering in Fourier space, where $y_\nu(\mathbf{q})$ and $\psi_\nu(q)$ are the Fourier transforms of $y_\nu(\mathbf{x})$ and $\psi_\nu(\mathbf{x})$, respectively. The quantity $w_\nu(\mathbf{b})$ is the the filtered map ν at the position $\mathbf{b}$. The *total filtered map* is the sum

$$w(\mathbf{b}) = \sum_\nu w_\nu(\mathbf{b}). \tag{3}$$

Therefore, the total filtered field is the result of two steps: a) filtering and b) fusion. Note that the combination in Eq. 3 is completely general.

The total filtered field w is *optimal* for the detection of the sources if

1. $w(\mathbf{0})$ is an *unbiased* estimator of the amplitude of the source, so $\langle w(\mathbf{0})\rangle = A$ (A is the amplitude of the point source at the chosen frequency of reference);

2. the variance of $w(\mathbf{b})$ is minimum, that is, it is an *efficient* estimator of the amplitude of the source.

If the profiles τ_ν (Gaussian beams assumed) and the frequency dependence of the point source f_ν are known and if the crosspower spectrum is known or can be estimated from the data, the solution to the problem is already known: the *matched multifilter* (MMF) [6]:

$$\mathbf{\Psi}(q) = \alpha\; \mathbf{P}^{-1}\mathbf{F}, \quad \alpha^{-1} = \int d\mathbf{q}\; \mathbf{F}^t\mathbf{P}^{-1}\mathbf{F}, \tag{4}$$

where $\mathbf{\Psi}(q)$ is the column vector $\mathbf{\Psi}(q) = [\psi_\nu(q)]$, $\mathbf{F}$ is the column vector $\mathbf{F} = [f_\nu \tau_\nu]$ and $\mathbf{P}^{-1}$ is the inverse matrix of the cross-power spectrum $\mathbf{P}$.

The frequency dependence f_ν is modelled in the following way:

$$I(\nu) = I_0 \left(\frac{\nu}{\nu_0} \right)^{-\gamma}, \tag{5}$$

where $I(\nu)$ is the flux at frequency ν, ν_0 is a frequency of reference, I_0 is the flux at that frequency of reference and γ is the *spectral index*. By using Eq. 5, the reference flux I_0 can easily be related to the reference amplitude A of the sources and the number of degrees of freedom is just one, the spectral index γ.

When we have the different simulated maps with the point sources, these images are iteratively filtered with different MMFs (in fact, we modify γ, but MMF depends on γ). The value of γ that maximises the SNR for a given source is an *unbiased* estimator of the real value of the spectral index of the source. After that, results are compared with the monofrequency *matched filter* (MF) [1, 9, 10].

The simulations that we use are a set of patches simulated from the Planck Sky Model, with the instrumental characteristics of the channels of the *Planck* mission at 44 (our frequency of reference) and 100 GHz, and lied far away from the Galactic plane. Then, point sources are added in such a way that in the same image we have sources with the same flux and spectral index. The threshold detection level is established at 5σ.

3 Results and discussion

We will compare the performance of the two methods in terms of the following aspects: spectral index estimation, source detection and reliability.

3.1 Spectral index estimation

In Fig.1 we see how the spectral indices are recovered by means of the MMF and by the MF. In general, we observe that the MMF is able to recover the value of γ with more accuracy and less uncertainty than the traditional MF.

Another aspect is that the error bars increase when I_0 is smaller. At $I_0 = 0.1$ Jy, we can see that the estimation of γ is not as good as we wish, because it has a great uncertainty. The main reason is that the signal to noise ratio is close to the threshold level we have imposed.

Finally, we can observe an interesting aspect of the matched filter. When we do not have the sufficient detections in at least one channel, the estimation of the spectral index is not good. In Fig. 1 we see that the spectral index is not well-estimated below $\lesssim 0.6$–0.7 Jy due to the Eddington bias [2] at those fluxes.

Due to these aspects, the matched multifilter is a suitable and effective tool to estimate the I_0 of the sources.

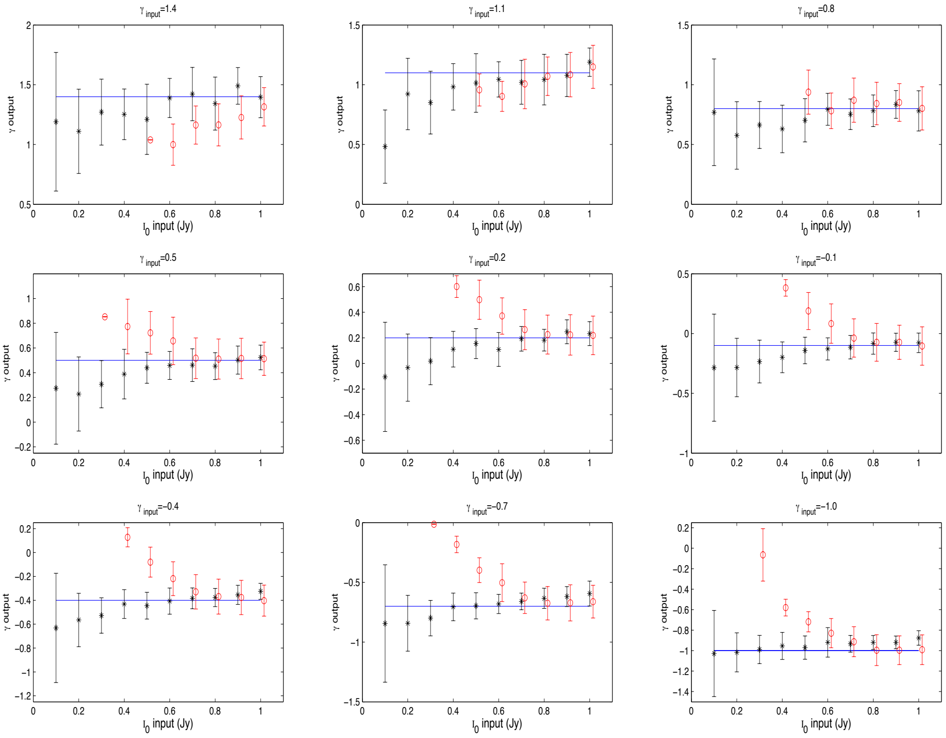

Figure 1: Values of γ recovered by means of the MMF (asterisks) and the MF (circles). The line indicates the ideal recovering of the input. The circles corresponding to the MF are slightly displaced in the horizontal axis in order to distinguish the results.

3.2 Reliability

In order to study the MMF in terms of reliability and spurious detections, we produce a new set of more realistic simulations with the following characteristics:

- We used as a background the same regions that in the previous subsection.
- The sources were simulated with an almost uniform Poissonian distribution (see [4] for more details about the method) at 44 GHz, with fluxes that follow the source number counts model of [3].
- The fluxes at 100 GHz were estimated assuming random spectral indices from the [5] distribution.
- The point source maps were filtered with the same resolution as the background maps and randomly added to them.

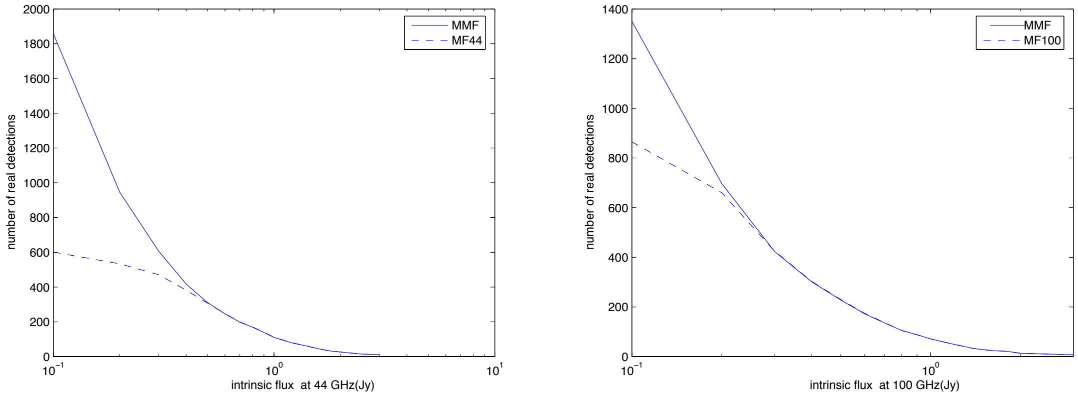

Figure 2: Number of real sources recovered by the MMF (solid line) and the MF (dashed line) at 44 GHz (left panel) and 100 GHz (right panel) whose intrinsic fluxes are higher that the corresponding value in the x axis.

There is an interesting quantity commonly used in the study of the performance of a source detector: the number of spurious sources. Spurious sources are fluctuations of the background that satisfy the criteria of the detection method and therefore are considered as detected sources. It is clear that the best method will be the one that has the best detections vs. spurious ratio. We have also changed the detection level from 5σ to 3σ in order to increase the number of spurious sources to make the analysis.

In Fig. 2 we observe the number of real sources that both methods are capable to detect, whose intrinsic fluxes are higher than the corresponding value in the horizontal axis. As we can see, MMF detects a higher number of real sources at low fluxes.

In Fig. 3 the *reliability* of both methods is compared. Reliability above a certain recovered flux is defined as $r = N_d/(N_d + N_s)$, where N_d is the number of real sources above that flux, and N_s is the number of spurious sources above the same flux. In general, we observe a higher reliability for the MMF at fluxes where the MF is not reliable anymore.

Acknowledgments

The authors acknowledge partial financial support from the Spanish Ministry of Education (MEC) under project ESP2004-07067-C03-01 and the joint CNR-CSIC research project 2008IT0059. LFL acnowledges the Spanish CSIC for a JAE-Predoc fellowship. Partial financial support for this research has been provided to JLS by the Spanish MEC and to JG-N by the Italian ASI (contracts Planck LFI Activity of Phase E2 and I/016/07/0 COFIS) and MUR. JG-N also acknowledges a researcher position grant at the SISSA (Trieste). ML-C acknowledges a post-doctoral fellowship from EGEE-III (FP7 INFSO-RI 222667). The authors are grateful to Raquel Fraga Encinas for reading carefully this document and her style and grammatical suggestions and acnowledge the use of the PSM, developed by the Component Separation Working Group (WG2) of the Planck Collaboration.

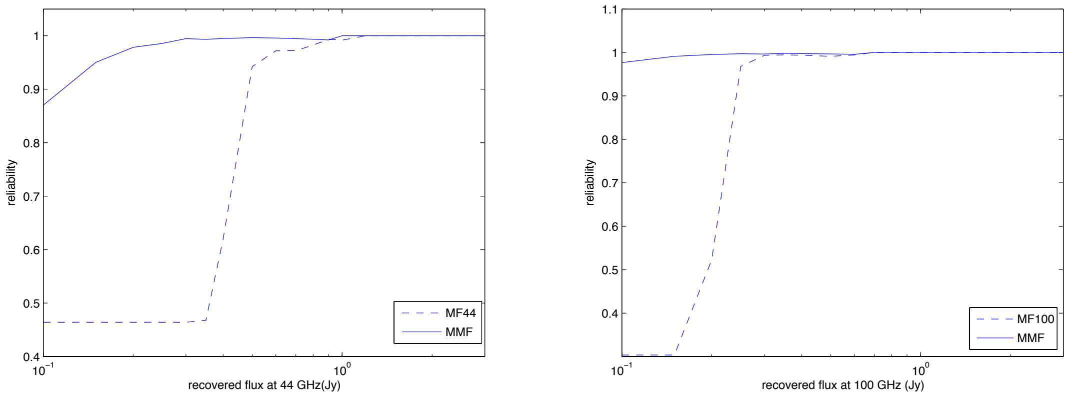

Figure 3: Reliability versus recovered flus for the MMF (solid line) and the MF (dashed line) at 44 GHz (*left panel*) and 100 GHz (*right panel*).

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