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Cheblet bases for galaxy modeling: a new tool for upcoming surveys

Y. Jiménez-Teja and N. Benítez

Instituto de Astrofísica de Andalucía (IAA-CSIC), Glorieta de la Astronomía s/n, 18008 Granada, Spain.

Abstract

The purpose of this work is to introduce a new mathematical tool to analyze image data coming from surveys. Built using Chebyshev polynomials and Fourier series, cheblet bases have shown to be highly powerful to model a huge variety of galaxies with different morphologies and features, displaying a better behavior than other well-know techniques as, for instance, the fitting by analytical profiles (Sersic, exponentials...) or shapelet bases. Several applications of this new method will be also displayed, as the use of the coefficients for the calculation of morphological parameters or the galaxy cluster modeling.

1 Introduction

During the last years, the improvements in the instrumentation and the technical advances have made it possible to obtain higher and higher quality data, what also requires more and more developed analyzing techniques. In the particular case of astronomical imaging, the improvement not only involves more accurate images, but also a larger amount of the data. In fact, an upward trend in large surveys is clearly observed in the recent proposals, so the necessity of efficient and precise computation techniques becomes evident.

Some of the widely-used methods in galaxy modeling lack some essential properties: either they are not capable of processing huge sets of data or they do not reach enough accuracy to fit every kind of galaxy. Cheblet bases arise as an excellent alternative, due to their linearity, flexibility, and capability of modeling with the same precision both the central bulge and the extended wings of the galaxies.

We will start summarizing the mathematical background where cheblets rely on to continue with a comparison of three methods for galaxy fitting, cheblets, GALFIT and shapelets. Then, some astrophysical applications of cheblet bases will be displayed, as the PSF deconvolution or the calculation of morphological parameters. We will finally draw some conclusions of this work.

2 Mathematical background

Cheblets basis functions are built in a separable way in polar coordinates, expanding the radial coordinate using Chebyshev rational functions [1] (which are Chebyshev polynomials mapped from the finite interval $[-1,1]$ to the semi-infinite one $[0,+\infty)$ by means of a rational function), and representing the angular component using Fourier series, due to its periodicity.

$$\{\phi_{n_1n_2}(r,\theta;L)\}_{n_1n_2} = \left\{\frac{C}{\pi}\, TL_{n_1}(r)e^{-in_2\theta}\right\}_{n_1,n_2}, \tag{1}$$

where $C = \begin{cases} 1, & \text{if } n_1 = 0 \\ 2, & \text{if } n_1 > 0 \end{cases}$ is a normalization factor, and L will be called the *scale* parameter from now on because it expresses the speed of the Chebyshev rational functions to reach their extrema. Basis functions are indexed by n_1, which is related to the order of the Chebyshev polynomial, and n_2, which means the Fourier frequency we are dealing with. It can be proved that this set of functions indeed constitutes an orthonormal basis of the Hilbert space of squared-integrable functions, $\mathcal{L}^2\left([0,+\infty)\times[-\pi,\pi]\,,\langle\cdot,\cdot\rangle\right)$, with a certain inner product, in such a way that any smooth enough function f can be decomposed in the following way:

$$f\,(r,\theta) = \frac{C}{2\pi^2}\sum_{n_2=-\infty}^{+\infty}\sum_{n_1=0}^{+\infty} f_{n_1,n_2}\, TL_{n_1}e^{in_2\theta} \tag{2}$$

and the coefficients of the decomposition, f_{n_1,n_2} will be called *cheblet coefficients.* Other mathematical properties can also be proved as, for instance, that the modulus of the coefficients shows an algebraic decay rate, which depends on the smoothness of the function f being represented. This property is of crucial importance for the practical application of this method, since it certifies the compactness of the basis and its reliability to efficiently represent galaxies using a finite number of coefficients. In order to show some visual information about these functions, cheblets with scale size $L = 1$ and coefficient indexes ranging from $0 \leq n_1 \leq 10$ and $-4 \leq n_2 \leq 4$ can be observed in Fig. 1.

3 Comparison of methods

In this section we will make a short comparison between cheblets and the two most used methods for galaxy fitting nowadays: GALFIT and shapelets. GALFIT sofware [3] is based on analytical profiles as Sersic, exponential, Gaussian or Moffat-Lorentzian functions. Its ability to represent galaxies with smooth profiles or regular morphologies is really great, but it does not model so efficiently irregular objects or spiral arms. Shapelets basis [2] are also built separably in polar coordinates, but using Hermite polynomials for the radial component and Gaussians for the angular one. This construction makes them highly suitable to represent small features present in the central bulge of the galaxies, but the exponential decay of wings

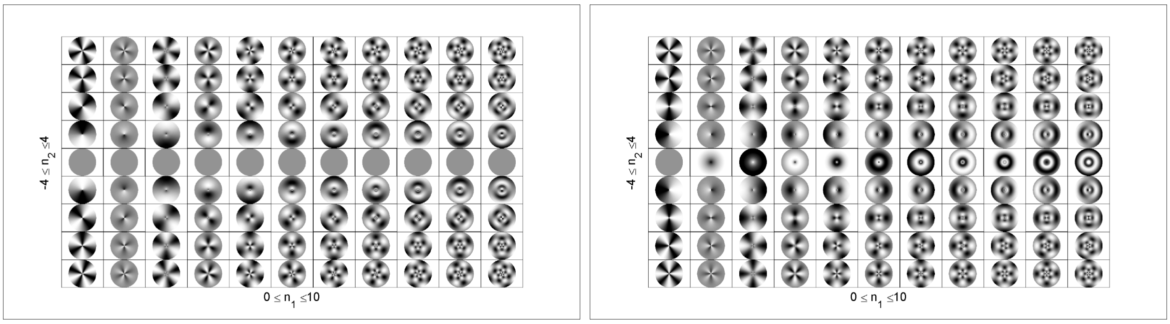

Figure 1: Real and imaginary components of the first Cheflet basis functions with scale size $L = 1$.

of these functions prevents them from modeling with the same accuracy the extended area of the objects. However, the fact than cheblets functions display a highly oscillating central zone, as shapelets do, and their wings tend to one as they approaching infinity, makes then capable to efficiently represent both the small features present in the image and the extended wings, and with the same accuracy.

4 Applications

In addition to the morphological modeling, cheblet decomposition can be used for many other applications as, for example, PSF deconvolution or morphological parameters calculation, among others.

The linearity property of cheblet bases produces the fact that the coefficients of the decomposition using the basis functions convolved convolved with a PSF coincide with coefficient of the decomposition of the original image as if this one were deconvolved:

$$f * \mathrm{PSF} = \left(\sum_{n_2=-\infty}^{+\infty} \sum_{n_1=0}^{+\infty} f_{n_1,n_2} \phi_{n_1 n_2} \right) * \mathrm{PSF} = \sum_{n_2=-\infty}^{+\infty} \sum_{n_1=0}^{+\infty} f_{n_1,n_2} \left(\phi_{n_1 n_2} * \mathrm{PSF} \right) \tag{3}$$

So, to perform the PSF deconvolution in a galaxy image it is just enough to convolve the original cheblet functions with the PSF and reconstruct the model which is get using these coefficients and the original basis functions (without being convolved with the PSF).

When it comes to morphological and photometric parameters, it can be shown that some of them can be obtained just using a few coefficients of the cheblet decomposition. If we take $p \geq 1$ and $R \in \mathbb{R}$ and calculate the generic integral:

$$\begin{aligned} I_p^{n_1} &= \int_0^R TL_{n_1}(r)\, r^p \; dr = 2 \sum_{j=0}^{n_1} \binom{n_1}{j} (-1)^j L^{-j/2} \frac{R^{p+j/2+1}}{2p+j+2} \cdot \\ & \cdot Re \left(e^{i n_1 \pi/2} i^{n_1+j} \, {}_2F_1 \left(n_1, 2p+j+2, 2p+j+3; \frac{-i\sqrt{R}}{\sqrt{L}} \right) \right), \end{aligned} \tag{4}$$

where ${}_2F_1$ is the hypergeometric function, then explicit expressions for the flux F, the unweighted centroid $x_c + iy_c$, the rms radius rms and the ellipticity ϵ can be easily obtained:

$$
\begin{aligned}
F(R) &= \int_0^R \int_{-\pi}^{\pi} f(r,\theta) r \, d\theta dr = 2\pi \sum_{n_1=0}^{+\infty} f^c_{n_1,0} \, I_1^{n_1} \\
x_c + i\, y_c &= \frac{1}{F} \int_0^R \int_{-\pi}^{\pi} f(r,\theta) r^2 (\cos\theta + i \sin\theta) \, d\theta dr = \\
&= \frac{2\pi}{F} \sum_{n_1=0}^{+\infty} \left[\left(f^c_{n_1,1} + f^c_{n_1,-1} \right) + i \left(f^s_{n_1,1} - f^s_{n_1,-1} \right) \right] \, I_2^{n_1} \\
rms &= \frac{F_{11} + F_{22}}{F} = \frac{2\pi}{F} \sum_{n_1=0}^{+\infty} f^c_{n_1,0} \, I_3^{n_1} \\
\epsilon &= \frac{F_{11} - F_{22} + 2iF_{12}}{F_{11} + F_{22}} = \frac{\sum_{n_1=0}^{+\infty} \left(f^c_{n_1,2} + f^s_{n_1,2} \right) \, I_3^{n_1}}{\sum_{n_1=0}^{+\infty} f^c_{n_1,0} \, I_3^{n_1}}
\end{aligned}
\tag{5}
$$

In this way, these parameters are calculated just using a few coefficients of the total decomposition of the images.

5 Conclusions

Cheblet bases have proved to be a highly reliable method to analyze galaxy images, displaying very desirable properties as flexibility, linearity and homogeneous precision in the results. They are in addition capable of solve those problems raised by other galaxy fitting techniques, as GALFIT or shapelets, which do not behave properly with either irregular nor extended objects. Moreover, cheblets are not only useful to efficiently reproduce the morphologies of the objects, but also to measure their photometry using just a few coefficients. Finally another astrophysical application as the PSF deconvolution can be easily performed due to the linearity of the bases, and their computational efficiency makes them highly suitable to be applied to large amount of data coming from surveys.

References

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[2] Massey, R., & Refregier, A. 2005, MNRAS, 363, 197

[3] Peng, C. Y., Ho, L. C., Impey, C. D., & Rix, H. W. 2002, AJ, 124, 266