Can the Tremaine - Weinberg method be used to derive the pattern speed of the bar and the spiral arms in the Milky Way?

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Abstract

The pattern speed of the non-axisymmetric structures in the galactic disc is a key parameter to understand the dynamics in the Milky Way. For none the Galactic bar nor the spiral arms is well determined as the current values have large uncertainties associated. We evaluate whether the Tremaine - Weinberg method as derived by Debattista et al. (2002) can be used to determine the pattern speed of the Galactic bar and the spiral arms in the Milky Way. We consider different situations; from simplistic test particle simulations with one structure to N-body simulations with both structures produced self-consistently. We also investigate Gaia mock catalogues with F0 and Red Clump stars as tracers. We conclude that this method can determine the pattern speed of the Galactic bar when going up to 6 kpc in the direction of the Galactic Center, whereas for the spiral arms all-sky radial velocity data up to 2-3 kpc is required.

1 Introduction

The structure of the Milky Way is far from being well understood, as there is still debate on the origin and nature of the different substructures (see \cite{2} for considerations on the Galactic bar and \cite{11} for the spiral structure).

To better constraint the nature of the non-axisymmetric structures of the Milky Way, that is, the bar and the spiral arms, we need to better determine their pattern speed. This
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velocity corresponds to the rotational velocity of the structure. There have been some previous measurements of such velocity, yielding values of $\Omega_{p,\text{bar}} \simeq 50 - 60$ km s$^{-1}$ kpc$^{-1}$ for the Galactic bar and $\Omega_{p,\text{sp}} \simeq 17 - 28$ km s$^{-1}$ kpc$^{-1}$ for the spiral structure (see [4]).

In 1984, Tremaine and Weinberg (T - W hereafter, [10]) derived a method to determine the pattern speed of barred external galaxies using only the surface density and the velocity along the line of sight. The importance of their method relies on its simplicity and its model independency. The unique geometry produced due to our position as observers within the Milky Way does not allow us to directly implement this method. Debattista et al. (2002) ([3]) derived a version of the T - W method to be used in our Galaxy in which they used the same assumptions as in [10], but using radial velocity data instead.

In order to do a good determination of the pattern speed, it is necessary to use radial velocity data with small uncertainties. Upcoming surveys such as Gaia and APOGEE (I & II) will determine the positions and velocities with unprecedented accuracy in the biggest volume of our Galaxy ever seen. Their data is expected to help unveil some of the mysteries of our Galaxy.

The main goal of this project is to evaluate whether the T - W method can be used to determine the pattern speed of the Galactic bar and the spiral arms of the Milky Way. We start applying the method to simplistic test particles simulations to more realistic N-body simulations. We also study Gaia mock catalogues to see whether RVS Gaia data will be suitable for the application of this method.

2 Tremaine - Weinberg method

The T - W method as derived by [3] follows the same assumptions as in the derivation made by [10], which are: the disk of the galaxy has zero thickness, the structure rotates as a solid rigid (that is, it has a well-defined pattern speed $\Omega_p$) and the surface brightness of the tracer obeys the continuity equation. In [3] they also assume that the tracer has to be dynamically relaxed.

From eq. (14) in [3] and assuming a circular orbit for the LSR, the pattern speed of a non-axisymmetric feature in a three dimensional non-inertial frame can be determined from:

$$\Omega_p = \frac{1}{R_\odot} \left( \frac{\sum_i f(r_i) \left( v'_{r,i} + \vec{v}_\odot \cdot \hat{r}_i \right)}{\sum_i f(r_i) \sin(l_i) \cos(b_i)} + V_{\text{LSR}} \right) = \frac{1}{R_\odot} \left( \frac{K}{P} + V_{\text{LSR}} \right)$$

(1)

where $R_\odot$ is the solar galactocentric radius, $v'_r$ is the heliocentric radial velocity, $\vec{v}_\odot \cdot \hat{r}_i$ is the radial peculiar motion of the Sun, $V_{\text{LSR}}$ is the tangential velocity of the LSR, $(l, b)$ are the galactocentric coordinates and $f(r)$ is a selection function that can be thought as the probability of detecting the tracer. We use two different functions: $f_1 = 1$ which can be understood as all stars are being detected and $f_2(r_{\text{star}} < r) = 1$ which corresponds to a step function for all stars inside a certain heliocentric radius $r$. 
Figure 1: Representation of the positions \((X, Y)\) for the model TWA10 (red dots) and the mock catalogues TWA10_RC (blue dots) and TWA10_RC_sbsm (green dots) in which we assume RC giants to be the tracers. The first mock catalogue contains all RC giants observed by Gaia with \(G_{RV,S} \leq 16.1\), whereas in the second mock catalogue the observational errors are constrained to be \(\sigma_\pi / \pi < 10\%\) and \(\sigma_{RV} \leq 10\) km s\(^{-1}\). The Sun is located at the position \((X,Y)_{\odot} = (-8.5,0)\) kpc.

3 Simulations and disk mock catalogues

In this study we use the test particle simulations of a Galactic bar described in [9] and the simulations of spiral arms described in [1]. We also use the N-body simulations B1 and B5 described in [7] and [8].

With these simulations we cover different physical cases; the test particle simulations contain one non-axisymmetric structure (either bar or spiral arms), whereas the N-body simulations contain both features produced self-consistently. Moreover, for each structure we use simulations with different physical parameters (imposed pattern speed, velocity dispersion, amplitude of the spiral arms and number of particles) to study if these affect the determination of the pattern speed.

In order to produce the mock catalogues, we assume two types of tracers: Red Clump giant stars for two test particles simulations (barR22 and TWA10) and F0 stars for a spiral arms simulation (TWA0). For the Galactic bar simulation, we produce a first catalogue considering only the interstellar extinction (doing a cut for \(G < 20\), barR22_G20) and two more catalogues with the expected Gaia errors (barR22_NOE with the real values and barR22_RCB with the error values). For each tracer in the spiral arms simulations we produce a mock catalogue using the expected Gaia errors (TWA0_F and TWA10_RC) and a second one from a subsample of them considering those particles with relatives errors in the parallaxes of \(\sigma_\pi \leq 10\%\) and in the radial velocities of \(\sigma_{RV} \leq 10\) km s\(^{-1}\). In Fig. [1] we represent the \((X,Y)\) positions for the Gaia mock catalogues produced for the spiral arm simulation TWA10.
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### Table 1: Summary of the values obtained for the mock catalogues

The columns describe, from left to right, the non-axisymmetric structure contained in the model, the tracer used, the pattern speed of the model $\Omega_{p,\text{sim}}$, the model or mock catalogue used, the errors applied, the recovered pattern speed $\Omega_{p,TW}$, the 75% CI and the number of particles used for the computation.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Tracer</th>
<th>$\Omega_{p,\text{sim}}$</th>
<th>Model</th>
<th>Errors applied</th>
<th>$\Omega_{p,TW}$</th>
<th>75% CI</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral arms</td>
<td>F stars</td>
<td>12</td>
<td>TWA0</td>
<td>$G \leq 20 + G_{\text{r vs}} \leq 16.1$</td>
<td>11.8, 11.9, 12.5</td>
<td>7.5 $\times 10^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TWA0 $F$</td>
<td></td>
<td>52.5, 48.3, 57.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TWA0 $F_{\text{subsm}}$</td>
<td></td>
<td>26.4, 26.2, 26.6</td>
<td></td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>RC giants</td>
<td>18</td>
<td></td>
<td>TWA10</td>
<td></td>
<td>17.9, 12.6, 19.2</td>
<td>1 $\times 10^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TWA10 $\text{RC}$</td>
<td>$G \leq 20 + G_{\text{r vs}} \leq 16.1$</td>
<td>18.5, 18.3, 18.6</td>
<td>3.5 $\times 10^6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TWA10 $\text{RC}_{\text{subsm}}$</td>
<td></td>
<td>30.3, 30.6, 30.6</td>
<td>$\sim 1 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Galactic bar</td>
<td>RC giants</td>
<td>50</td>
<td>barR22</td>
<td></td>
<td>66.8, 50.9, 92.3</td>
<td>6 $\times 10^6$</td>
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<td></td>
<td>barR22 $\text{G20}$</td>
<td>Extinction $G \leq 20$</td>
<td>26.0, 26.01, 26.04</td>
<td>$\sim 26.3 \times 10^6$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>barR22 $\text{NOE}$</td>
<td>$G_{\text{r vs}} \leq 15$ (real)</td>
<td>24.952, 24.941, 24.953</td>
<td>$\sim 4.4 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>barR22 $\text{RCB}$</td>
<td>$G_{\text{r vs}} \leq 15$ (errors)</td>
<td>24.953, 24.948, 24.960</td>
<td>$\sim 4.4 \times 10^6$</td>
<td></td>
</tr>
</tbody>
</table>

*Units: km s$^{-1}$ kpc$^{-1}$*

### 4 Gaia mock disk catalogues

Table 1 contains a summary of the determined pattern speed and its uncertainty for each mock catalogue when using the selection function $f_1$ in which we consider all the stars in the catalogue. From left to right, the columns correspond to the feature contained in the simulation, the tracer used, the imposed pattern speed, the name of the catalogue and its characteristics, the recovered pattern speed and its uncertainty range and the number of particles used.

The only catalogue for which the method is capable of recovering the imposed pattern speed is the one corresponding to the Red Clump giants for spiral arms (TWA10 $\text{RC}$). In the case of the F0 stars in the spiral arm simulation, the low brightness of the tracer and the cut in magnitude due to the introduction of the Gaia errors greatly reduces the size of the mock catalogue, thus not having enough particles to correctly determine the pattern speed. Hence, we rule out F0 stars as tracers for RVS Gaia data.

In the case of the Red Clump mock catalogues, we do a more thorough study by applying the selection function $f_2(r)$ in which we consider all particles in the catalogue up to a certain heliocentric distance.

We represent the absolute error between the imposed pattern speed and the one recovered by the method as a function of the heliocentric distance for each mock catalogue in Fig. for the bar and in Fig. for the spiral arms models. For the Galactic bar, it can be seen that the pattern speed is recovered when only the interstellar extinction is considered, but not when the Gaia errors are introduced. This indicates that Gaia data will not be suitable to determine the pattern speed of the bar when using Red Clump giants as tracers. The divergence observed is due to the term in the denominator in eq. 1 going to zero, $P \rightarrow 0$. This term represents the asymmetry in the density for that simulation and it can cancel if the overdensities due to the non-axisymmetric structure have equal weight at each side of the...
Figure 2: Absolute error between the imposed pattern speed ($\Omega_{p,\text{sim}}$) and the recovered pattern speed ($\Omega_{p,TW}$) using the selection function $f_3$ up to $r_{\text{lim}} \in [3, 7]$ kpc for the model barR22 and the mock catalogues barR22_G20, barR22_NOE and barR22_RCB. The dashed line corresponds to an absolute error of 2 km s$^{-1}$ kpc$^{-1}$.

simulated galaxy.

For the spiral arms, the recovered pattern speed for the reduced mock catalogue (TWA10_RC_sbsm) presents a systematic offset with respect to the imposed one. As the pattern speed is recovered for the complete catalogue (TWA10_RC), we recommend not doing subsamples for a certain relative error, as it may introduce a systematic error that will be difficult to quantify.

5 Discussion

The work presented here is further described in [5]. From the results presented we conclude that the T- W method as derived by [3] can be used on Gaia Red Clump radial velocity data to determine the pattern speed of the spiral arms under certain conditions:

- The sample has to be large. The minimum number of particles required to recover the pattern speed in our simulations was $N_{\text{min}} \simeq 10^6$ particles.

- The selection function, or how is your tracer detected due to absorption, spatial distribution, instrumental resolution, etc, needs to be well constrained.

The spiral arms require all-sky radial velocity data up to only to 2 – 3 kpc. If instead of using the selection function $f_2(r)$ for all 360 degrees we restrict it to the Galactic center, we find that, the pattern speed of the bar can be recovered when going up to, at least, 6 kpc. Given that Gaia will only reach up to 5 kpc into the Galactic center, it will be necessary to use radial velocity data from the upcoming high resolutions spectroscopic surveys like APOGEE-I/II and 4MOST to determine the pattern speed of the Galactic bar.
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![Graph](image)

Figure 3: Absolute error between the imposed pattern speed ($\Omega_{p,\text{sim}}$) and the recovered pattern speed ($\Omega_{p,TW}$) using the selection function $f_3$ up to $r_{lim} \in [3, 7]$ kpc for the model TWA10 and the mock catalogues TWA10_RC and TWA10_RC_sbsm. The dashed line corresponds to an absolute error of 2 km s$^{-1}$ kpc$^{-1}$.

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References