Highlights of Spanish Astrophysics VIII, Proceedings of the XI Scientific Meeting of the Spanish Astronomical Society held on September 8–12, 2014, in Teruel, Spain. A. J. Cenarro, F. Figueras, C. Hernández-Monteagudo, J. Trujillo Bueno, and L. Valdivielso (eds.)

Inconsistencies in the harmonic analysis applied to pulsating stars

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Abstract

Harmonic analysis is the fundamental mathematical method used for the identification of pulsation frequencies in asteroseismology and other fields of physics. Here we introduce a test to evaluate the validity of the hypothesis in which Fourier theorem is based: the convergence of the expansion series. The huge number of difficulties found in the interpretation of the periodograms of pulsating stars observed by CoRoT and Kepler satellites lead us to test whether the function underlying these time series is analytic or not. Surprisingly, the main result is that these are originated from non-analytic functions, therefore, the condition for Parseval's theorem is not guaranteed.

1 Introduction

The unexpected huge number of frequencies found in multi periodic stars and the ubiquitous presence of correlated noise in the residuals of the fitting of the light curves of these stars, are some of the numerous difficulties found in the interpretation of the periodograms in asteroseismology.

The necessary condition for a correct Fourier analysis, i.e. the convergence of the Fourier expansion, is guaranteed when the function is analytic, otherwise Fourier analysis is not a consistent approximation. In this work we examine the differentiability of the function describing photometric data of pulsating stars obtained by *CoRoT* and *Kepler*. We determine how smoothly the data points of the light curve can be fitted in order to fully reproduce the function. We call this fine structure property the connectivity of the function at a given point. We studied this property by means of two numerical approaches: cubic splines - analytic, and autoregressive moving average (ARMA) [1] methods - non-analytic.

2 Analyticity test: connectivity

To perform the differentiability analysis of the underlying function our approach is to study how compactly connected the discrete data are through a property called connectivity. Note that we are studying the differentiability of the continuous function underlying the discrete data and not the discrete data themselves. The connectivity $\mathcal{C}_n$ of a data point x_n is defined as

$$C_n = \epsilon_n^f - \epsilon_n^b \tag{1}$$

with ϵ_n^f, ϵ_n^b defined as

$$\begin{aligned} \epsilon_n^f &= x_n^f - x_n \\ \epsilon_n^b &= x_n^b - x_n \end{aligned}$$

where x_n^f and x_n^b are forward and backward extrapolations respectively made from the data bracketing a datapoint x_n. From these equations the numerical approximation of the point derivative $\mathcal{D}_n$ at x_n can be expressed through the connectivity as:

$$\mathcal{D}_n = \frac{\mathcal{C}_n + x_{n+1} - x_{n-1}}{2\Delta t} \tag{2}$$

which reduces to the typical point derivative for discrete data when $\mathcal{C}_n = 0$. When connectivities are not zero but independent randomly distributed values, the derivative is still well-defined. In that case, the connectivities can be considered simply as deviations from the derivative at this point. Otherwise, the derivability condition is not fulfilled.

The connectivity is similar to the non-differentiability coefficient introduced by Wiener [5] but in our case extrapolations are introduced to calculate the coefficient.

In order to to calculate the extrapolations the method requires a model that allows to approximate arbitrarily well any analytic function. The Stone-Weierstrass theorem [3] states that a function uniformly continuous in a closed interval can be approximated arbitrarily well by a polynomial of degree n. Therefore, provided a sufficient amount of data, a spline function should provide a good extrapolation for the datapoint - i.e. the residuals form an independent random sequence (white noise). We also make use of an ARMA approach which is capable of fitting non-analytic functions. Both results (ARMA and splines) are then compared.

3 HD 174936

We first applied the method to the light curve of the multi periodic δ Scuti star HD 174936 observed by the *CoRoT* instrument, using the 422 independent frequencies found in its periodogram [2]. These are distributed in a range of frequencies below 900 μHz. This very large range of excited frequencies is not predicted by any non-adiabatic model describing opacity-driven pulsators.

The numerical methods described above were applied to an analytic model of this star. The time series of this model was calculated using the amplitudes and phases of the

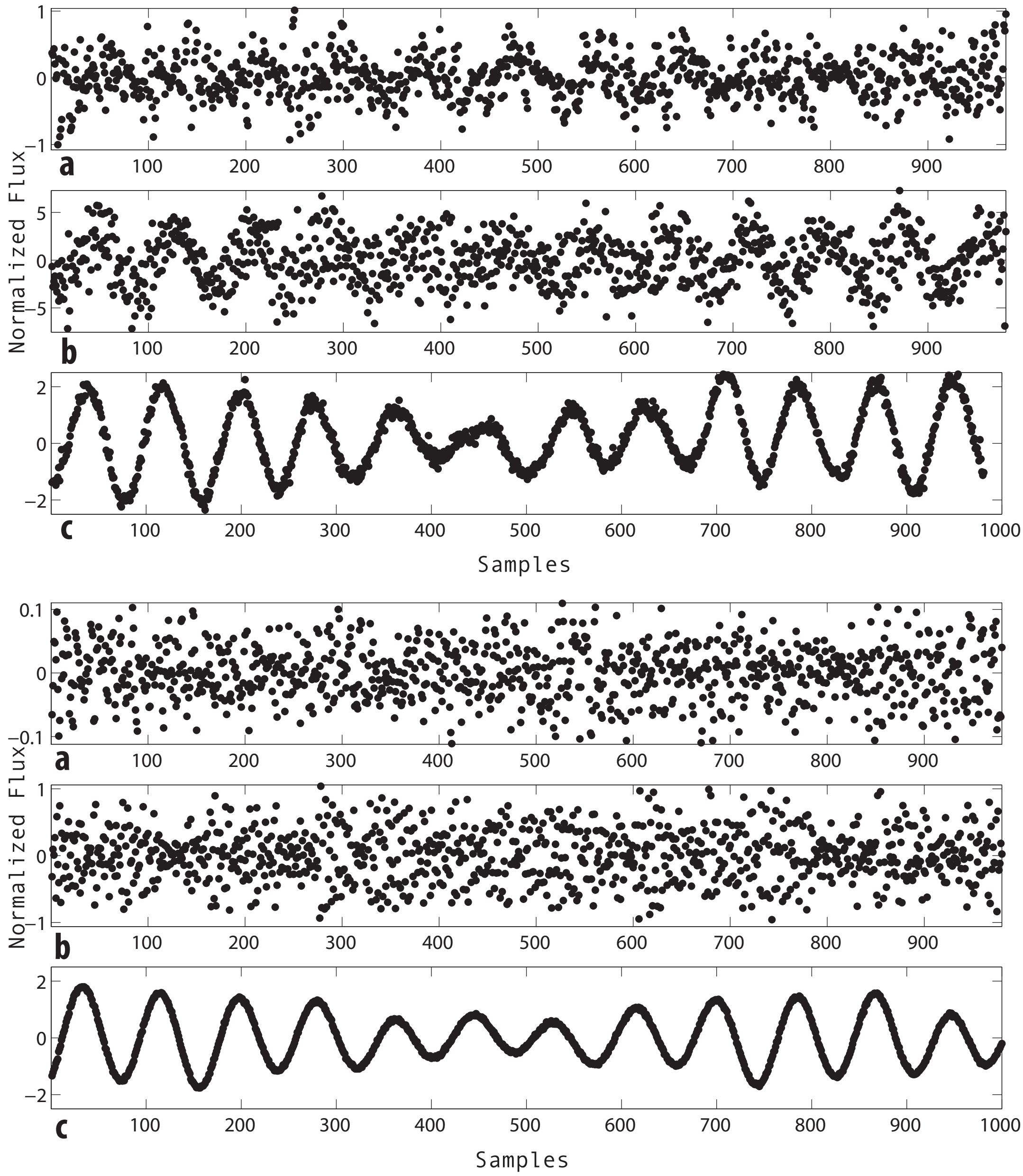

Figure 1: Connectivities of the *CoRoT* data for the δ Scuti star HD 174936 (above) and the connectivities of an analytic model based on the first 422 frequencies detected in the light curve of HD 174936 using classical Fourier techniques (below). **a**, ARMA connectivities. **b**, cubic splines connectivities. **c**, the original light curve. (Note the different scaling in top and middle panels of both figures.)

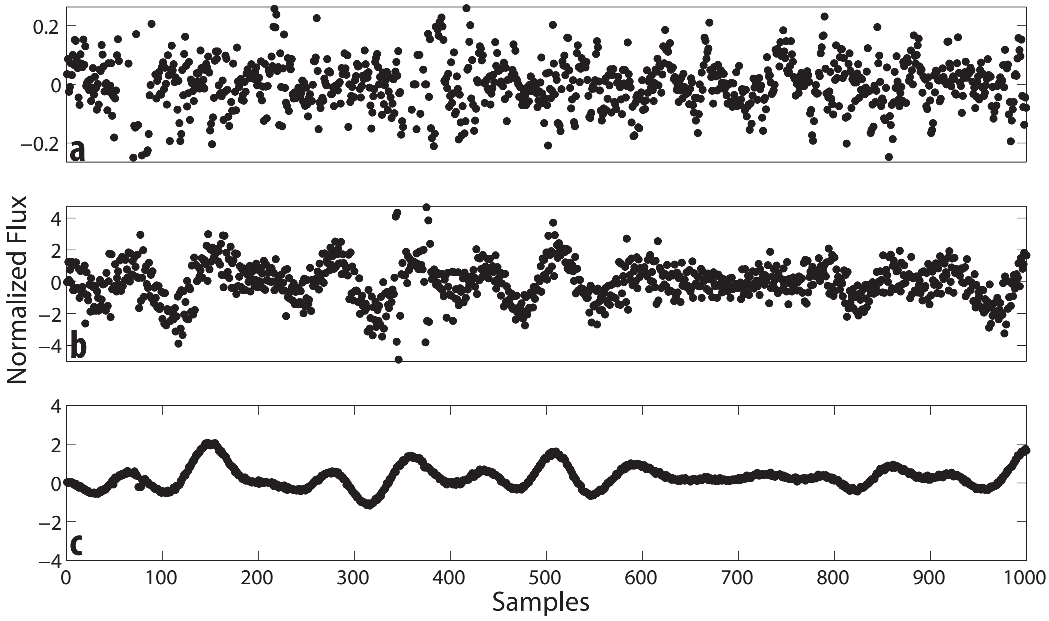

Figure 2: Connectivities of the *Kepler* data for the hybrid star KIC 006187665 and the original light curve for comparison. **a**, ARMA connectivities. **b**, cubic splines connectivities. **c**, *Kepler* light curve for comparison. (Note the different scaling in top and middle panels.)

frequencies found in [2] for the signal component, and a noise component based on additive gaussian white noise with a level obtained from the same reference.

For both the observed data and the time series of the analytic model (Fig. 1), the connectivities calculated using cubic splines have higher dispersion than the ARMA ones. Nevertheless, a striking effect is seen in the observed data that can not be noticed in the analytic model: spline connectivities are not randomly distributed but strongly correlated with the signal, suggesting that these data are not sufficiently well described by the analytic model.

4 KIC 006187665

In order to discard possible instrumental effects specifically linked to a given instrument, a similar test was applied to the time series from another star KIC 006187665 observed by the *Kepler* satellite, and classified as a hybrid Gdor/Dscuti pulsator in [4]. The classical Fourier analysis of the light curve of this star supplied by the *Kepler* satellite yielded 659 significant peaks.

The connectivities of this light curve show a very similar behaviour to the *CoRoT* ones (Fig. 2), that is, cubic splines connectivities show a more dispersed distribution than the ARMA connectivities, and strongly correlated with the original *Kepler* time series.

The results of this chapter the previous one have been confirmed by calculating the Pearson correlation coefficients for both splines and ARMA connectivities of the two light curves studied.

5 Conclusions

The test introduced here through the connectivities allow us to fully characterise the differentiability of the underlying function of a time series. Although the underlying function can be non-analytic due to the noise itself being non-analytic, our test assumes the non-analyticity of this component and evaluates the deterministic component through the correlation of the connectivities.

The tests carried out here show that the underlying function describing the light variations of some pulsating stars is non-analytic. This implies that the conditions for the Parseval theorem might be not satisfied. Therefore, it is not guaranteed that the signal can be represented by a Fourier series and thereby the periodogram could be not a consistent estimator of the frequency content of the underlying function.

This inconsistency in the application of harmonic analysis of pulsating stars might be extended to any physics domain in which the convergence of the Fourier expansion is not evaluated with a consistent test as the one we introduce here.

Acknowledgments

JPG and RG acknowledge support from the "Plan Nacional de Investigacin" under project AYA2012-39346-C02-01, and from the "Junta de Andalucía" local government under project 2012-P12-TIC-2469. JCS acknowledges support from MINECO and FEDER funds through the Astronomy and Astrophysics National Plan under number AYA2012-39346-C02-01. Also acknowledges support from the European project SpaceInn with reference 312844 within the european SPACE program FP7-SPACE-2011-1, and also acknowledges funding support from the Spanish Ministerio de Economía y Competitividad under Ramón y Cajal subprogram.

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