Estimating all-sky night brightness maps from finite sets of SQM measurements

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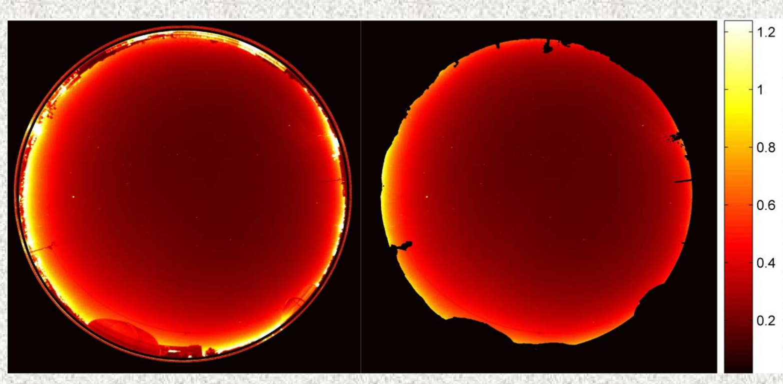
Abstract: The all-sky night brightness distributions recorded at observing sites with moderate to high levels of light pollution can be efficiently described by polynomial series or relatively low order. This opens the way for estimating these continuous distributions from discrete sets of measurements made in different directions of the sky with photometric detectors of low spatial resolution as, e.g. the Sky Quality Meter, SQM™ (10º HWHM).

Modal estimations of the night sky brightness can be obtained by expanding their equal-area projection maps as a series of orthonormal functions, in particular Zernike polynomials, and fitting the unknown modal coefficients to the measurements provided by the detector.

Least squares and minimum variance estimators can be easily developed once the linear functional relationship between the measurements and the actual sky brightness distribution is established.

All-sky night brightness maps in ZEA projection



AstMon™ B-band image taken at UCM Astronomical Observatory Nov. 11th 2012, 21:30:54 UT. Brightness scale in mJy/arcsec²

Zenithal Equal-Area projection of spherical coordinates (azimuth α, zenith angle z) onto the Zernike unit-radius circle (r,θ) :

$$\theta = \alpha \quad ; \quad r = \sqrt{2}\sin\left(\frac{z}{2}\right)$$

Zernike polynomial expansion of the night sky brightness:

$$B(\mathbf{r}) = \sum_{k=1}^{\infty} a_k Z_k(\mathbf{r}) \quad \longrightarrow \quad B'(\mathbf{r}) = \sum_{k=1}^{M} \hat{a}_k Z_k(\mathbf{r})$$

$B(\mathbf{r})$: actual

$B'(\mathbf{r})$: estimated

SQM FOV

- Hemispherical $\boldsymbol{\alpha}(z,\alpha)$

$$FOV_s(\boldsymbol{\alpha}) = K\cdot C\left[2\delta(\boldsymbol{\alpha};\boldsymbol{\alpha}_s)/\pi\right]\exp\left[-\gamma\,\delta(\boldsymbol{\alpha};\boldsymbol{\alpha}_s)^2\right]$$

with $\delta(\boldsymbol{\alpha};\boldsymbol{\alpha}_s) = \cos^{-1}\left[\cos(z)\cos(z_s)+\sin(z)\sin(z_s)\cos(\alpha-\alpha_s)\right]$

- In the ZEA domain $\mathbf{r}(r,\theta)$

$$P_s(\mathbf{r}) \equiv FOV_s[\boldsymbol{\alpha}(\mathbf{r})] = K\cdot C\left[\frac{2L(\mathbf{r};\mathbf{r}_s)}{\pi}\right]\exp\left[-\gamma\,L(\mathbf{r};\mathbf{r}_s)^2\right]$$

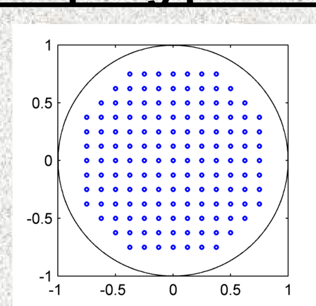
with $L(\mathbf{r};\mathbf{r}_s) = \cos^{-1}\left[(1-r^2)(1-r_s^2)+\left(r\sqrt{2-r^2}\right)\left(r_s\sqrt{2-r_s^2}\right)\cos(\theta-\theta_s)\right]$

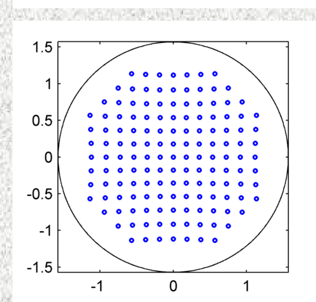
SQM measurements model

$$m_s = \mu_s + \nu_s = \sum_{k=1}^{\infty} a_k \left[\int_C P_s(\mathbf{r})\, Z_k(\mathbf{r})\, \mathrm{d}^2\mathbf{r}\right] + \nu_s$$

s: index of the sampling point ν_s: measurement noise

Example of sampling patterns





Top: Square sampling grid in the ZEA domain, with 145 sampling directions >15º above the horizon; Bottom: Corresponding sampling directions in azimuthal coordinates

Linear least-squares estimator with no *a priori* knowledge of the night sky brightness statistics

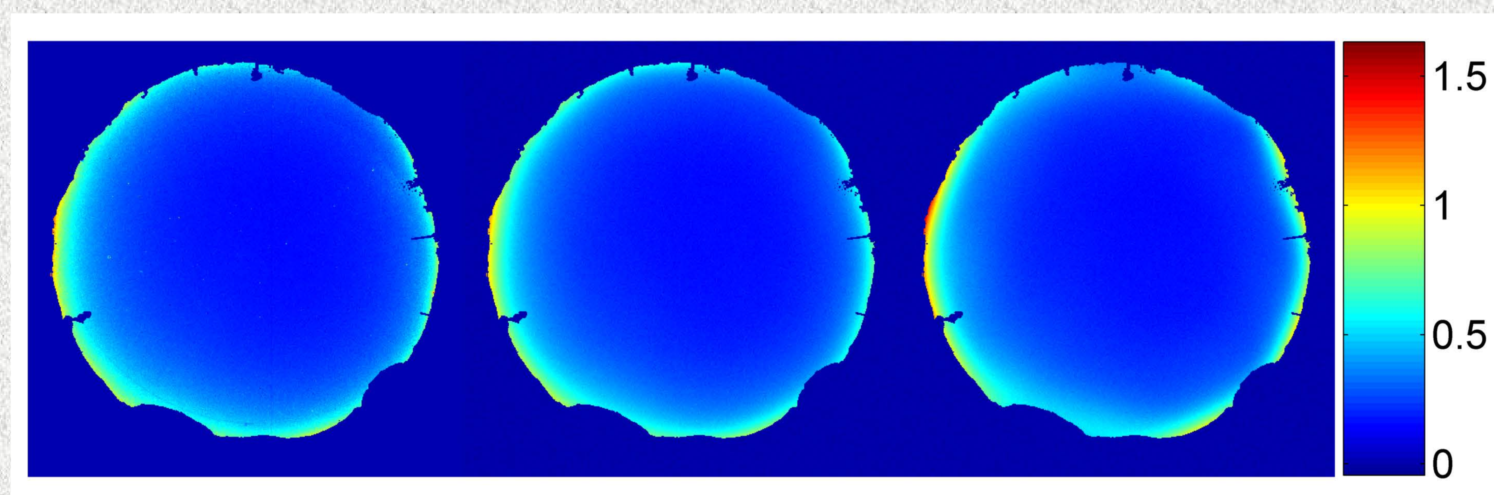
$$\hat{\mathbf{a}} = \mathbf{R}\mathbf{m} \quad \text{with} \quad \mathbf{R} = \left(\mathbf{Z}_M^T\,\mathbf{Z}_M\right)^{-1}\mathbf{Z}_M^T$$

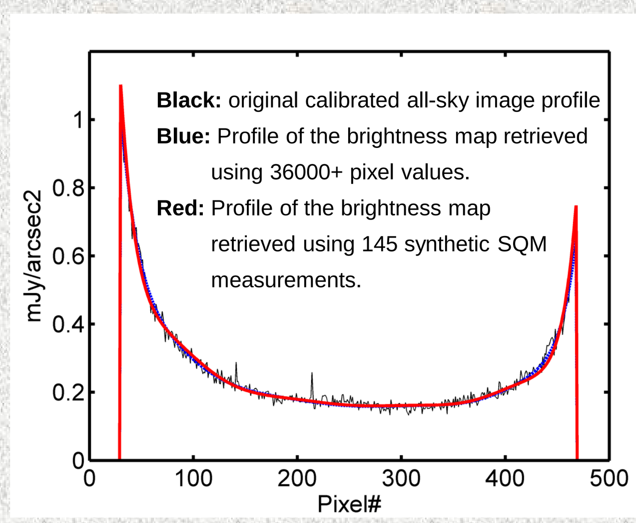
$\mathbf{R}$: Least-squares (LSQ) estimator (pseudoinverse of $\mathbf{Z}_M$)

$$(\mathbf{Z})_{sk} = \int_C P_s(\mathbf{r})\, Z_k(\mathbf{r})\, \mathrm{d}^2\mathbf{r}$$

The elements of $\mathbf{Z}_M$ are the ZEA FOV-weigthed integrals of the modes.

Example of reconstruction using synthetic measurements





Up Left: original calibrated all-sky image. Up Center: brightness map with 66 modes retrieved by LSQ using 36000+ pixel values for the fit. Up Right: brightness map with 66 modes retrieved by LSQ from 145 synthetic measurements with SQM FOV. Random noise with the same variance as the original has been added to the reconstructed maps.

Conclusions

- Photometric detectors with low spatial resolution are useful devices for estimating the continuous all-sky night brightness distributions.
- The number of degrees of freedom of a typical all-sky night brightness map recorded at sites with moderate levels of light pollution is relatively small.
- A small-sized set of brightness samples taken at different sky directions shall be enough to reconstruct the whole maps with reasonable accuracy and precision.

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