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A likelihood ratio test to assess the significance of source detections in imaging observations.

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Abstract

A new method to estimate the significance of point source observations carried out by Imaging Atmospheric Cherenkov Telescopes is presented. It is based on a likelihood ratio test procedure under the assumption of a well-known PSF and a smooth background. The method is tested with Monte Carlo simulations based on real observations and its sensitivity is compared to standard methods widely used in IACT Astronomy which do not incorporate PSF information.

1 Introduction

The statistical significance of an observation is a key issue in *signal starved* fields such as Imaging Atmospheric Cherenkov Telescopes (IACT) Astronomy. The classical paper of Li&Ma [1], proposed a robust and reliable method of estimating that significance which has become the standard in the field. Since at that time gamma-ray instrumentation had very limited angular resolution, the method was designed as an event counting technique which does very little use of the instrument resolution, given by its Point Spread Function (PSF), and background distribution. Therefore, the achieved sensitivity should be lower than the one of methods that incorporate that information. On the other hand an important feature of the *Li&Ma* method is that it uses not only ON observations (data at the source position), but also OFF data, (observations of a control region). This removes the need of assuming a certain behaviour of the background in the source position.

Li&Ma is a particular case of techniques based on maximum likelihood principles, these kind of methods, such as [2] and [3], are usually difficult to implement. This explains why their use in IACT Astronomy is still restricted to special analysis, such as sky maps [4], or spectral line studies in Dark Matter searches [5]. Nevertheless, they are usually more sensitive than event counting methods, at the risk of loosing robustness.

In this article, we describe a technique that takes into account our knowledge of the instrumental PSF in gamma-ray observations, under the assumption of a smooth background.

It can be understood as a generalization of the *Li&Ma* method or a particular application of the one proposed in [4] to a specially relevant case: the search for point sources. We show how this *PSF-Likelihood* method can recover more information about the source of interest, keeping at the same time the simplicity of the standard *Li&Ma* method. We test it on a realistic situation, using a set of *toy Monte Carlo* samples generated using real background and data from observations of the Crab Nebula performed by the MAGIC experiment ([6]).

1.1 Maximum Likelihood with background estimation

IACTs operate in harsh environments and their performance is highly dependent on the atmospheric and observing conditions. As a consequence the background affecting an observation is highly variable and it is usually estimated jointly with the signal. We will assume the simplest case in which the result of an observation is a set of two one-dimensional θ^2 histograms, showing the number of events detected as a function of the squared angular distance θ to the real or control source positions (ON and OFF histograms). It can be easily generalized to more complex cases. If the statistics are enough, the number of events per bin will follow a Poisson distribution, leading to the Binned Likelihood function for each histogram:

$$\mathcal{L}(X|\Theta) = \prod_{i}^{N} \frac{f_i^{n_i}(\Theta) e^{-f_i(\Theta)}}{n_i!} \tag{1}$$

where Θ is the parameter space for our model, i is the bin number (for a total of N bins), n_i the number of events in bin i and f_i the value of the test model in the given bin. It is common and convenient to work with the negative logarithm of this function, $L(X|\Theta) = -\log \mathcal{L}(X|\Theta)$. When ON and OFF are considered the total likelihood function is the sum of the terms for both histograms.

The method is then based on computing the likelihood ratio for two different hypothesis, λ and its logarithm, $-2\log\lambda \equiv 2\{\mathbf{L}'_{H0}(X|\Theta) - \mathbf{L}'_{H1}(X|\Theta)\}$.

From [7] it is known that $-2\log\lambda$ asymptotically follows a χ^2_r distribution for large event counts, where r is the difference in the number of degrees of freedom between the hypotheses. This can be used to compute the probability of the observed excess being due to a background fluctuation. It can also be translated into a significance TS, defined as $TS = \pm\sqrt{\chi^2_1}$, where χ^2_1 is the value of the χ^2 with one d.o.f. corresponding to the same probability as the original χ^2_r.

The *Null hypothesis, H0* states that ON and OFF observations have the same origin, with only a normalization factor due to possible differences in the effective observation time and therefore the background model can explain both ON and OFF observations equally well. One the other hand the *Alternative hypothesis, H1* affirms that ON and OFF have different origins. OFF contains only background events, while ON contains also signal events.

The *Li&Ma* method In the *Li&Ma* method, $r = 1$, only one bin is defined in each, ON and OFF regions. Their size must be fixed *a priori*. They are usually selected as the ones giving the optimal discrimination taking into account the PSF of the instrument and the expected background statistics. Then TS has an analytic expression, which is normally known as the *Li&Ma* formula (17). A variant of this method that uses a background modeling to improve the estimation of the number of events in the OFF region is called *Li&Ma with fitted off* , it usually provides an improved sensitivity.

The *PSF-Likelihood* method Here we are not only interested in the amount of ON and OFF events, but also in the way they are distributed. Our *Alternative hypothesis, H1* states that the second component needed to explain the ON data is distributed around the source position following the PSF. Both histograms are fitted at the same time with models derived from each hypothesis. The total log-likelihood $\mathbf{L}'(X|\Theta)$ for each hypothesis can be used to compute the likelihood ratio $-2\log\lambda$ and the associated TS.

2 Method and results

The case we considered was the search for a point source. In order to test the method, we created a pipeline in `ROOT` (see [8]). For each simulation, the software created θ^2 histograms for the source and background based on real MAGIC observations of the Crab Nebula, and an OFF region. A background fit was performed to the later to smooth it and remove spurious fluctuations arising from the finite sample statistics. Both templates, together with their fitted models, are shown in Figure 1. For the PSF model , after considering other possibilities a simple the Gaussian PSF with a fixed σ was finally selected.

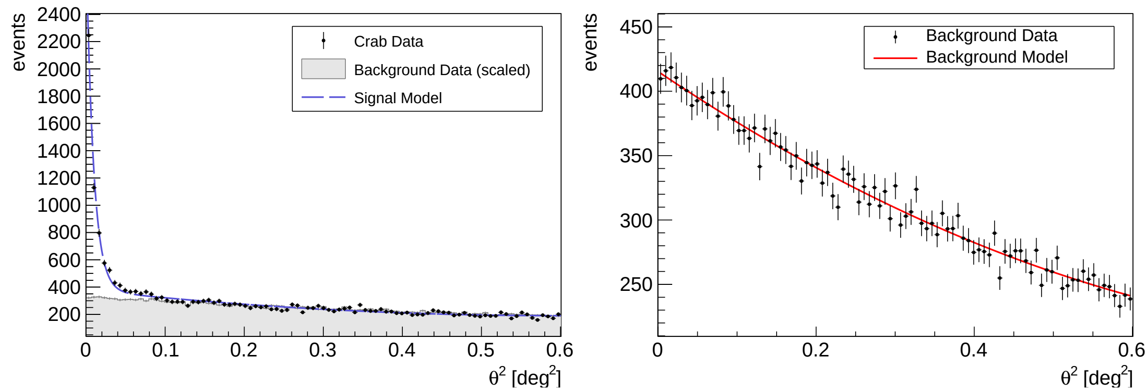

Figure 1: Experimental θ^2 distribution measured over Crab Nebula and Background samples with the MAGIC telescopes. We find that simple gaussian and 2nd degree polynomial models fit reasonably well both distributions, with a total $\chi^2 = 63.4$ for 88 $d.o.f.$ in the background sample. ($\chi^2/d.o.f. = 16.3/28$ in the $\theta^2 < 0.2$ range).

Using the isolated Crab signal and the background template, $3 \cdot 10^7$ simulated ON and OFF samples were generated for 10 different signal fractions: 0%, 0.2%, 0.5%, 1%, 2%, 3%, 5%, 8%, 15% and 50%, covering a wide range of signal strengths. The test was performed for 1, 3 and 5 OFF regions. In each simulation, the statistical hypotheses H0 and H1 were tested with the ON and OFF samples for the *Li&Ma* , *Li&Ma with fitted off* and *PSF-Likelihood* . The significances were calculated and saved to a text file for further analysis. An example of the intermediate result can be seen in Figure 2.

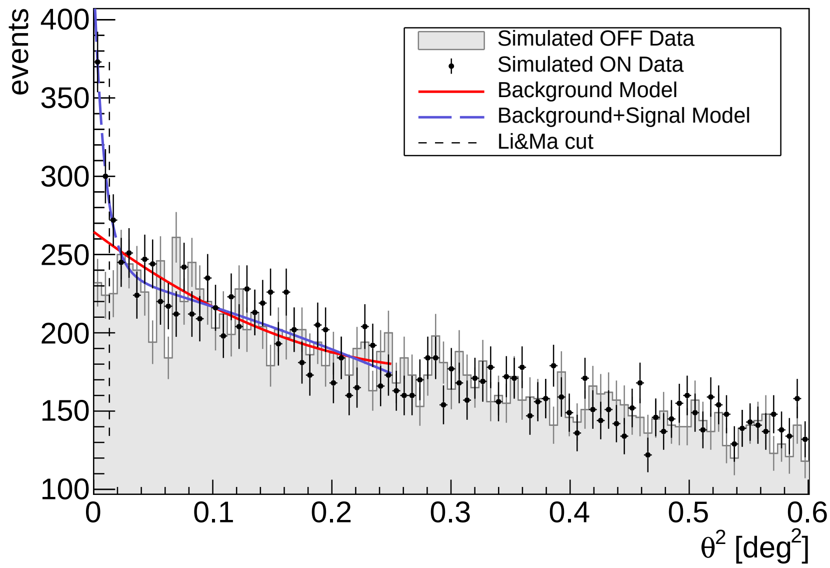

Figure 2: Testing the method with simulated samples for an ON region with 8% excess events (black points) and a single OFF region (shaded region). Background-only model (red line) and background+signal model (green line) are shown as reference. The dashed vertical line shows the region used by the *Li&Ma* method.

The main concern when testing a new detection method is its statistical correctness. The resulting distribution of significances on pure background samples should follow the expected probability distribution, which for the methods described here is a χ^2_1 according to [7]. From Figure 3, we see that all the methods give statistically correct results when tested against background samples with random statistical fluctuations.

Next, the method is tested against existing ones. In Figure 3, the different methods are compared in detail for a sample containing a 3% of signal events. Figure 4 shows the results on different fractions of excess events. From the figure, it seems evident that the performance not only depends on the background statistics, but also on the method itself. In that sense, *PSF-Likelihood* outperforms *Li&Ma* and *Li&Ma with fitted off* for every step in the signal fraction, meaning that taking into account the PSF also contributes to improve the sensitivity of the method.

It must be noted that the method allows to compute the number of events detected from the source, which is simply related to the normalization factor of the fitted PSF and its uncertainty. This procedure can also be translated to data with finer energy bins, i.e. the source spectrum. The PSF to use is then the appropriate one for this particular energy range. In that case, the improved sensitivity of the method translates in smaller uncertainties for the number of excess events, and therefore the fluxes. For example, for an increase in the

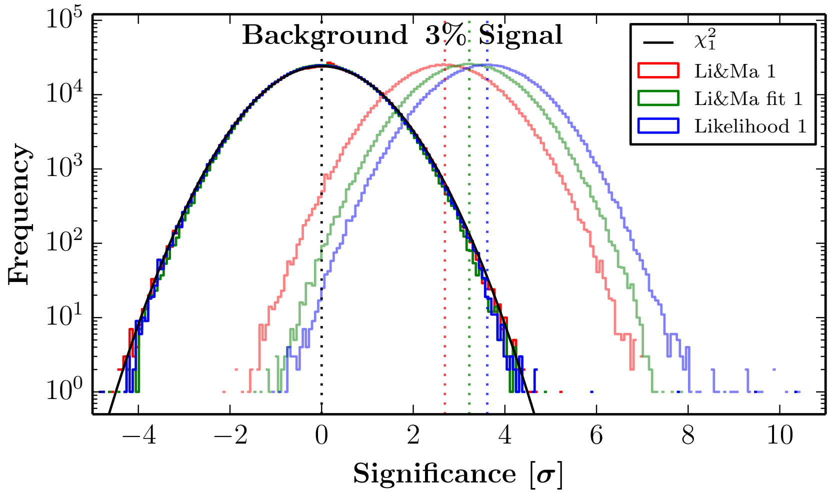

Figure 3: Significance calculated over background-only Monte Carlo simulations and over a sample with 3% signal. The expected χ^2_1 distribution for the background-only case is plotted as reference.

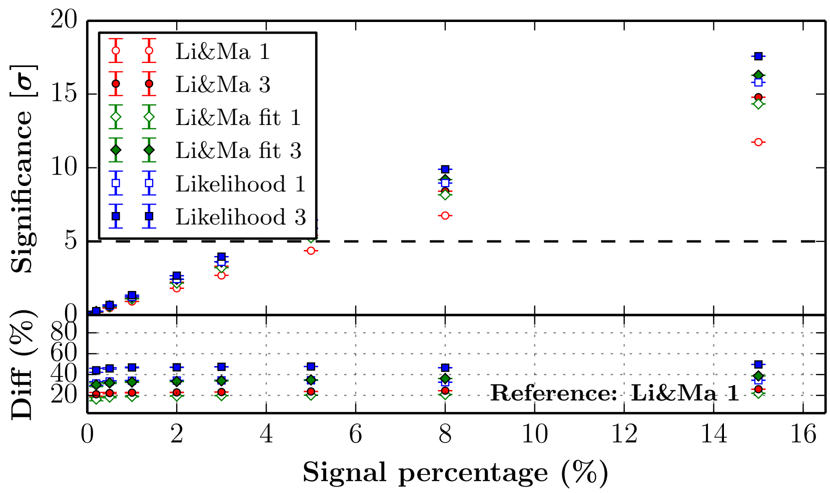

Figure 4: Significance of different methods for Monte Carlo simulations. For each method, we test the '1 OFF region' and the '3 OFF region' cases. The 5σ limit (the usual detection threshold) is drawn as an horizontal dashed line.

significance of $\sim 20\%$ against *Li&Ma* method, the decrease in the statistical uncertainties of the fluxes is found to be of the order of $\sim 15 - 30\%$, depending on the signal strength.

3 Conclusions

We have described a possible implementation of the Binned Likelihood Ratio method to estimate the significance of astronomical observations of point-like sources in IACT observations. Our implementation uses the known PSF of the instrument and combines the ON and OFF observations in an elegant yet straightforward way, enabling us to improve the sensitivity by increasing the number of OFF positions.

When tested on Monte Carlo simulated observations with OFF data, we found that it reproduces the expected χ^2_1 significance distribution, which means that the chance probability of a false detection is correctly estimated. On the other hand, when a certain amount of signal is introduced, an improvement in sensitivity is found over other methods. Part of the improvement is found to be due to the increased effective background statistics of the fit, but a significant fraction of it stems from the inclusion of the PSF.

The method can be easily generalized to include additional information such as the 2D shape of the PSF, which should increase the discriminating power. It could also be used for other imaging observations of point-like sources that incorporate an independent background observations.

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