Highlights of Spanish Astrophysics VII, Proceedings of the X Scientific Meeting of the Spanish Astronomical Society held on July 9 - 13, 2012, in Valencia, Spain. J. C. Guirado, L. M. Lara, V. Quilis, and J. Gorgas (eds.)

CHEF applications to the ALHAMBRA survey

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Abstract

CHEF bases (from Chebyshev and Fourier) [3] have been developed as a mathematical tool especially design to model astronomical images, particularly, galaxies. They consist on a set of mathematical bases built in polar coordinates using Chebyshev rational functions to represent the radial component and Fourier series to expand the angular coordinate. They have proved to be highly precise and very compact, yielding much better results than other widely used techniques as GALFIT or the shapelet bases, for instance. Several practical applications for the CHEFs have been implemented further from the morphological fitting, as for example, the calculation of photometric and morphological parameters (such as the flux, the centroid, the ellipticity, etc.), the generation of realistic samples of mock galaxies with complex structures, the subtraction of bright galaxies in clusters, the estimation of the shear for weak lensing studies and the measurement of the photometry in lensing arcs. This contribution will be devoted to present the results yielded by the application of the CHEFs to the data from the ALHAMBRA project, showing not only the successful results obtained but also the capability, velocity and efficiency of the algorithm to work with the large set of data coming from this survey.

1 Introduction

The CHEF bases [3] were developed with the aim of solving the two main problems that affected the modeling in astronomical imaging: the proper fitting of every kind of morphology, reaching the same precision both in the central bulge and the wings of extended sources, and the creation of an independent and automated algorithm capable of processing huge amount of data without nearly any interaction from the user. Composed in a separable way in polar coordinates using both Chebyshev rational functions and Fourier series, the CHEF functions have been proved to constitute an orthonormal basis of the Hilbert space of the finite energy functions. So any smooth function can be decomposed as a linear combination of these functions:

$$f\left(r,\theta\right)=\frac{C}{2\pi^2}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}f_{nm}\,TL_n\left(r\right)W_m\left(\theta\right) \tag{1}$$

where W_m represents both the $\sin(m\theta)$ and the $\cos(m\theta)$ functions and the so called CHEF coefficients f_{nm} are calculated by

$$f_{nm} = \frac{C}{2\pi^2} \int_{-\pi}^{\pi} \int_{0}^{+\infty} f\left(z, \phi\right) TL_n\left(z\right) \frac{1}{z+L} \sqrt{\frac{L}{z}} W_m\left(\phi\right)\ dz\ d\phi. \tag{2}$$

The minimax properties of the Chebyshev polynomials are inherited by their by-products, the Chebyshev rational functions, so they are much more efficient when fitting the usual galaxy profiles needing less coefficients than the Hermite polynomials (used in the shapelet approach). For this reason, CHEF have shown to be very compact displaying a fast decay rate in the modulus of the CHEF coefficients.

The comparison of the CHEFs with other widely known fitting techniques as GALFIT [5, 6] or shapelets [7] has stated the better performance of our methods. While GALFIT is incapable of efficiently reproduce the complex structures and small features present in some real images, the orthonormality and flexibility of the CHEFs allow them to model all these morphological characteristics up to the noise level. As for shapelets, the use of Gaussians in their construction limits the precision in models of extended sources, since the cut-off of the Gaussians generally bounds the light flux coming from the disks. As a result, shapelets do not usually describe the galaxies with the same accuracy in the central bulge and the extended wings. However, the natural shape of the Chebyshev polynomials makes them catch all the light present in the image, so the same precision is reached in all the areas.

We will focus now on the different practical applications we have implemented for the CHEFs beyond the usual morphological analysis, which have been applied to the data from the ALHAMBRA project [4, 1]. They can be summarized on the determination of analytical formulae for some photometric and morphological parameters and the optimization of an algorithm to obtain a precise photometry.

2 Analytical parameters determination

The first task we carried out with the CHEFs and the ALHAMBRA data was obviously modelling every object present in the images (but the stars, which were masked in a preprocessing step) using a Python code that we have written with this aim. The results of the application of the code on one of the ALHAMBRA fields, can observed in Fig. 1, where it is specially remarkable the image composed by the CHEF models of all the galaxies, totally free of noise.

Once we had this noise-free, accurate models, the immediate step was extracting useful information from them. For this reason, we developed analytical formulae to get some photomteric and morphological parameters just using the CHEF coefficients. We obtained expressions for the flux, the centroid, the rms radius, and the ellipticity [3]. Let us define the

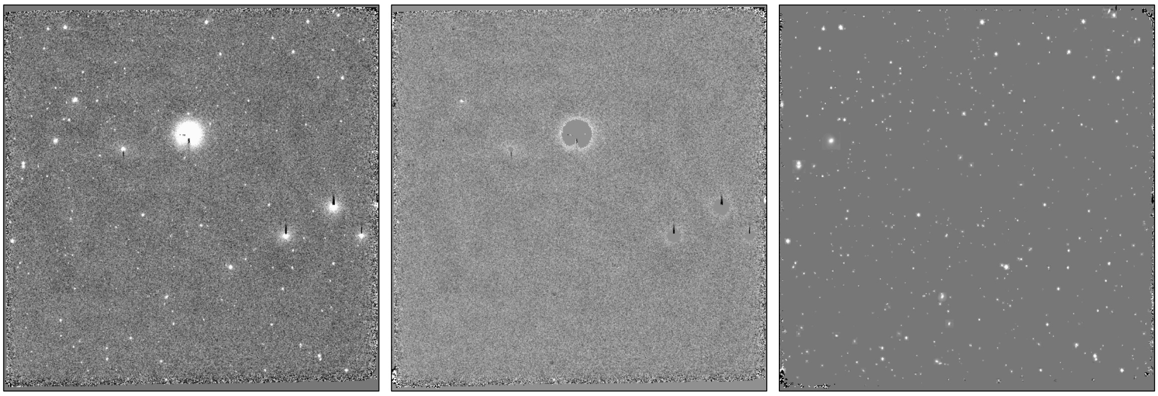

Figure 1: Processing of an ALHAMBRA field by CHEFs: the original image appears in the left panel, the middle one shows the residuals after the subtraction and the image composed with all the CHEF models can be observed in the right image

expression:

$$I_q^n = 2\sum_{j=0}^{n}\left(\begin{array}{c} n \\ j \end{array}\right)(-1)^j L^{-j/2}\,\frac{R^{q+j/2+1}}{2q+j+2}\;Re\left(e^{in\pi/2} i^{n+j}\;{}_2F_1\left(n, 2q+j+2, 2q+j+3; \frac{-i\sqrt{R}}{\sqrt{L}}\right)\right),$$

for $n > 0$, being ${}_2F_1$ the hypergeometric function. If $n = 0$, then simply

$$I_q^0 = \frac{R^{q+1}}{q+1}.$$

Then, the previously mentioned parameters can be calculated with the CHEF coefficients by means of:

$$\begin{aligned}
F(R) &= \int_0^R \int_{-\pi}^{\pi} f(r,\theta) r \; d\theta dr = 2\pi \sum_{n=0}^{+\infty} f_{n,0}^c \, I_1^n, \\
x_c + i\, y_c &= \frac{1}{F}\int_0^R \int_{-\pi}^{\pi} f(r,\theta) r^2 (\cos\theta + i \sin\theta) \; d\theta dr = \\
&= \tfrac{2\pi}{F} \textstyle\sum_{n=0}^{+\infty} \left[\left(f_{n,1}^c + f_{n,-1}^c\right) + i\left(f_{n,1}^s - f_{n,-1}^s\right)\right] \, I_2^n, \\
r_{rms} &= \frac{F_{11}+F_{22}}{F} = \frac{2\pi}{F}\sum_{n=0}^{+\infty} f_{n,0}^c \, I_3^n, \\
e &= \frac{F_{11}-F_{22}+2iF_{12}}{F_{11}+F_{22}} = \frac{\sum_{n=0}^{+\infty}\left(f_{n,2}^c + f_{n,2}^s\right)\, I_3^n}{\sum_{n=0}^{+\infty} f_{n,0}^c \, I_3^n}.
\end{aligned} \tag{3}$$

(For the two latter we have used their expressions according to the quadrupoles moments F_{ij}). However, these formulae works in the case of ideal, totally noise free data, so we have implemented a Bayesian approach to optimize the calculation of the flux in real observing conditions.

3 Magnitude measurements by CHEFs

The CHEF algorithm uses SExtractor [2] code to detect the sources. However, SExtractor tends to overestimate the background, so we did not use it to remove the background. Instead of that, we estimated the local background level around each galaxy using the residuals of the corrsponding CHEF model outside the KRON ellipse of every object. Then, we used the formulae for the flux described in the Sect. 2 to calculate an initial value for the photometry. As it was previously commented, this expression is optimal for an ideal, noise free case, so we decided to slightly modify it to improve the accuracy. Applying a Bayesian approach it can be proved that the optimal estimator for the flux can be obtained after using the following correction weights:

$$F_{opt} = \left[\frac{\sum_{ij} m_{ij}}{\sum_{ij} \frac{m_{ij}^2}{n_{ij}^2}} \right] \frac{m}{n^2} F, \tag{4}$$

where m its corresponding CHEF model and n the map noise. To test this algorithm, we first create a sample of 350 mock galaxies with Sersic profiles with indexes ranging from 0.5 to 4 and sheared with different levels of elipticity from 0 to 0.5. The Gaussian noise was added and the photometry was compared with the one obtained by the shapelets. The results, displayed in [3], showed that the CHEFs performed much better in these simple conditions, reaching a maximum error of 1.6% against the 13.5% yielded by the shapelets. In addition, our results were perfectly homogeneous, without any bias because of the ellipticity of the clumpiness of the objects. However, we wanted to test the photometric pipeline on a more realistic set of data, so we built a database of noise free models using the galaxies form the UDF and adapted the to the observational characteristic of ALHAMBRA (pixel scale, zero point, photometric and morphological interpolation according to the filter wavelength, and convolution with an ALHAMBRA PSF). We then inserted these models at randomly chosen locations inside an ALHAMBRA field and with random rotation angles.

This complex sample was used to measure the photometry with the CHEFs and compare it to the one obtained by SExtactor. Notice that database of analytical models from the UDF provides us with enough information to know the real magnitude of the objects, so we compared it with the two measured. The results are displayed in Fig. 2. As it can be observed, SExtractor tends to underestimate the photometry, due to both its inaccurate estimation of the background and the way it measures the flux, discarding all the flux outside a certain aperture. However, the CHEFs do not show any kind of bias and they behave really well up to faint magnitudes. Although the scatter in our case is higher, this is perfectly reasonable since we are measuring total magnitudes instead of aperture ones.

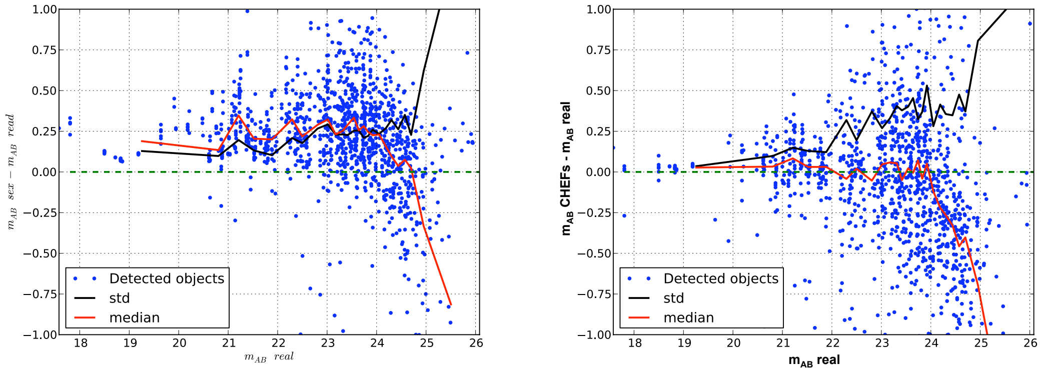

Figure 2: Comparison of the real magnitudes for the realistic sample of galaxies, with the measured by SExtractor (left panel) and by the CHEFs (right).

4 Conclusions

We have described some of the applications we have developed for the particular case of the ALHAMBRA survey. We have not only modeled all the objects present in the images but we have also calculated some of their main morphological and photometric parameters. We have especially worked on the estimation of the photometry, to obtain the most accurate results in the presence of noise, introducing a Bayesian corrector factor. We have tested this photometric pipeline first using simple analytical profiles and comparing the result to the ones yielded by the shapelets. Our results have turned to be nearly one order of magnitude more precise than these. We have later created a much more realistic, complex sample of galaxies, using the objects from the UDF as a base. After adapting them to the observational characteristics of ALHAMBRA, we have measured their photometry in two different ways, both using the CHEFs and SExtractor code. After comparing the results with the real, analytical photometry of the objects, we conclude that SExtractor obtain biased measurements, and to to underestimate the magnitudes, while the CHEFs are much more accurate, with homogeneous estimations and just showing a higher dispersion since they are not measuring inside any aperture.

References

[1] Benítez, N., et al. 2009, ApJ, 692, 1, L5

[2] Bertin, E. & Arnouts, S. 1996, A&A Suppl, 117, 393

[3] Jiménez-Teja, Y. & Benítez, N. 2012, ApJ, 745,150

[4] Moles, M., et al. 2008, AJ, 136, 1325

[5] Peng, C.Y., Ho, L.C., Impey, C.D., & Rix, H.W. 2002, AJ, 124, 266

[6] Peng, C.Y., Ho, L.C., Impey, C.D., & Rix, H.W. 2010, AJ, 139, 2097

[7] Refregier, A. 2003, MNRAS, 338, 35