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Kelvin-Helmholtz and Rayleigh-Taylor instabilities in partially ionised prominences

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Abstract

We study the modification of the classical criterion for the linear onset and growing rate of the Kelvin-Helmholtz Instability (KHI) and the Rayleigh-Taylor instability (RTI) in a partially ionised plasma in the two-fluid description. The plasma is composed of a neutral fluid and an electron-ion fluid, coupled by means of particle collisions. The governing linear equations and appropriate boundary conditions, including gravitational terms, are derived and applied to the case a single interface between two partially ionised plasmas. For high collision frequencies and low density contrasts the KHI is present for super-Alfvénic velocity shear only. For high density contrasts the threshold velocity shear can be reduced to sub-Alfvénic values. For the particular case of turbulent plumes in prominences, we conclude that sub-Alfvénic flow velocities can trigger the KHI thanks to the ion-neutral coupling, but with long time scales. Ion-neutral collisions have a strong impact on the RTI growth rate, which can be decreased by an order of magnitude compared to the value in the collisionless case. The time scale for the development of the instability is much longer than in the classical incompressible fully ionised case. This result may explain the existence of prominence fine structures with life times of the order of 30 minutes.

1 Introduction

Solar prominences are among the best candidates to display significant deviations from one-fluid Magnetohydrodynamics, since they are relatively cold objects whose ionisation degree is not well measured, but is assumed to be around 50%. The basic structure of prominences is a lump of chromospheric plasma prevented from falling due to gravity or evaporating due to thermal conduction by the magnetic field. There is not a clear picture of the precise morphology or magnetic field configuration in these intriguing objects (see for example the reviews [1, 2]).

The Kelvin-Helmholtz Instability (KHI) is a well-known magnetohydrodynamic instability that arises at the interface between two fluids in relative motion (see the classical textbooks by [3, 4]). It is known that a velocity shear at the interface between two incompressible plasmas is always unstable in the absence of a magnetic field. The presence of a magnetic field component along the flow direction suppresses the KHI for sub-Alfvénic velocity shear in fully ionised plasmas. However, in a partially ionised plasma composed of ions and neutrals, the two species behave very differently when a magnetic field is applied. Neutrals are insensitive to the magnetic field so that their natural tendency is to be unstable for any velocity shear, while ions feel the stabilising presence of the magnetic field. The coupling between neutrals and ions through collisions is therefore crucial for the growth rate of the KHI and its evolution. The KHI in partially ionised incompressible plasmas has been studied under different contexts. For example [5] studied the KHI due to shear flow at the interface between two incompressible and partially ionised plasmas concluding that ion-neutral collisions cannot suppress the instability of neutrals for sub-Alfvénic velocity shear.

It is well stated in hydrodynamics that a lighter fluid below a heavier one is subject to the Rayleigh-Taylor instability (RTI for short). If no magnetic field is present the configuration is always unstable if the heavier fluid is on top of the lighter one. The magnetic field stabilises perturbations up to a critical wavenumber along its direction, while it has no effect on perturbations perpendicular to it (which are thus unstable) [3]. The classical treatment of the RTI uses the incompressible assumption. However, to consider the effect of compressibility provides a more realistic description of the behaviour of astrophysical plasmas in general and solar plasmas in particular. A considerable effort has been done in studying the RTI in realistic configurations. It is interesting to investigate the RTI beyond the MHD theory in a two fluid configuration, even in simple geometrical models, and test the modifications of the simple formulae before including those effects in 3D computational models.

2 Multifluid equations

The general transport equations for a multi-component plasma can be derived from the Boltzmann kinetic equation, with some assumptions about the collisions terms [6]. MHD deals only with the plasma as a single fluid, so no effects of partial ionisation are ever considered at this level. The first extension from MHD is to consider only the modifications due to collisions in the generalised Ohm's law and energy transport. A different approach beyond single-fluid MHD is to use a two-fluid treatment, in which ions and electrons are considered together, as an ion-electron fluid, and neutrals are considered separately.

In the following expressions the subscripts i, n and e stand for ions, neutrals and electrons, respectively. We write the momentum equations of neutrals and ions plus electrons, and we neglect the electron inertial terms because of the small electron mass compared to the other species. The variables have their usual meanings, while α_{in} is the ion-neutral friction coefficient. We need a generalised Ohm's law and induction equation. These are obtained from the momentum equation of electrons by neglecting again their inertial terms. We neglect the magnetic diffusion terms and notice that the gravity term is constant (so its curl vanishes), obtaining a simple form of the induction equation. The mass conservation equations are not modified by the presence of gravity. In addition, we assume adiabatic energy equations for all the species. Hence, our system of basic equations is

$$\begin{aligned}
\rho_i\left(\frac{\partial\vec{v}_i}{\partial t}+\vec{v}_i\cdot\nabla\vec{v}_i\right) &= -\nabla p_{ie}+\frac{1}{\mu}\left(\nabla\times\vec{B}\right)\times\vec{B}-\rho_i\vec{g}-\alpha_{in}\left(\vec{v}_i-\vec{v}_n\right),\\
\rho_n\left(\frac{\partial\vec{v}_n}{\partial t}+\vec{v}_n\cdot\nabla\vec{v}_n\right) &= -\nabla p_n-\rho_n\vec{g}-\alpha_{in}\left(\vec{v}_n-\vec{v}_i\right)\\
\frac{\partial\vec{B}}{\partial t}+\vec{v}\cdot\nabla\vec{B} &= \nabla\times\left(\vec{v}_i\times\vec{B}\right),\\
\frac{\partial p_{ie}}{\partial t}+\vec{v}_i\cdot\nabla p_{ie} &= -\gamma p_{ie}\nabla\cdot\vec{v}_i,\qquad \frac{\partial p_n}{\partial t}+\vec{v}_n\cdot\nabla p_n=-\gamma p_n\nabla\cdot\vec{v}_n,\\
\frac{\partial\rho_i}{\partial t}+\nabla\cdot\left(\rho_i\vec{v}_i\right) &= 0,\qquad \frac{\partial\rho_n}{\partial t}+\nabla\cdot\left(\rho_n\vec{v}_n\right)=0.
\end{aligned}\tag{1}$$

The friction coefficient can be further expressed in terms of the ion-neutral and neutral-ion collision frequencies ($\nu_{\rm in}$ and $\nu_{\rm ni}$) by means of $\alpha_{in}=\rho_n\nu_{\rm in}=\rho_i\nu_{\rm ni}$. We keep $\nu_{\rm in}$ as a free parameter for later use. We also define the total density $\rho=\rho_n+\rho_i$, the neutral fraction $\eta_n=\rho_n/\rho$ and the ion fraction $\eta_i=\rho_i/\rho$, with $\eta_n+\eta_i=1$. Hence, the parameter η_n indicates the ionisation degree, from $\eta_n=0$ for a fully ionised plasma to $\eta_n=1$ for a neutral gas.

Next, we study linear perturbations from the uniform state. We consider the normal mode decomposition and write the temporal dependence of the perturbation as $e^{-i\omega t}$. We Fourier analyse in the spatial directions where the medium is uniform and write the perturbations as $e^{ik_x x+ik_y y}$, with k_x and k_y the wavenumbers in the x and y-directions, respectively, and $\vec{k}=k_x\hat{x}+k_y\hat{y}$ the wavenumber parallel to the surface.

3 Kelvin-Helmholtz Instability

For this part of the study, the gravity is assumed to be negligible, but there are equilibrium velocities parallel to the boundary in each region. We first check that in the collisionless plasma the ion-electron fluid behaves as in the classical problem as has exactly the same instability threshold, while the neutral fluid is always unstable. The growth rate is lower than the predicted by the classical formulae because of compressibility, and increasing the ratio k_y/k_x we approach to the incompressible case.

We consider next the coupling between ions and neutrals through collisions. Our purpose is to assess the effect of ion-neutral collisions on the results of the collisionless case. We express the collision frequency, $\nu_{\rm in}$, in units of the surface wave frequency, $\omega_{\rm k}=k_z\left((B_1^2+B_2^2)/(\mu\rho_{\rm i1}+\mu\rho_{\rm i2}\right)^{1/2}$. We compare the results when we increase $\nu_{\rm in}/\omega_{\rm k}$. In the panels of Fig. 1 we plot the results for three different values of $\nu_{\rm in}/\omega_{\rm k}$ with a density contrast of $\rho_2/\rho_1=2$. Different line styles are used depending on the value of k_y/k_z. For comparison

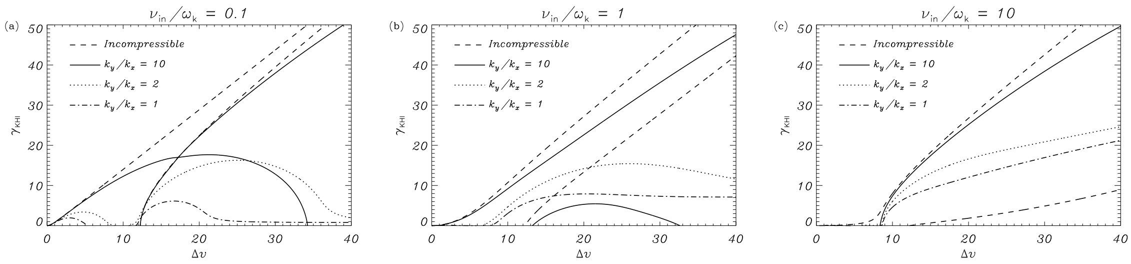

Figure 1: Dimensionless growth rate, $\gamma_{\rm KHI}$, as a function of the dimensionless velocity shear at the interface, Δv, in a collisional plasma with (a) $\nu_{\rm in1}/\omega_{\rm k} = \nu_{\rm in2}/\omega_{\rm k} = 0.1$, (b) $\nu_{\rm in1}/\omega_{\rm k} = \nu_{\rm in2}/\omega_{\rm k} = 1$, and (c) $\nu_{\rm in1}/\omega_{\rm k} = \nu_{\rm in2}/\omega_{\rm k} = 10$. The meaning of the different lines is indicated within the panels. In all computations $\rho_2/\rho_1 = 2$, $v_{\rm ai}/c_{\rm si} = 5$, $c_{\rm si} = c_{\rm sn}$, $\zeta_{\rm n1} = \zeta_{\rm n2} = 0.5$, and $k_z L_z = \pi$.

purposes we have also plotted using dashed lines the equivalent growth rates of the incompressible case. We obtain that for $k_y/k_z \gg 1$ the growth rates of the incompressible limit are recovered. The results displayed in Fig. 1 show a complicated evolution of the unstable modes when $\nu_{\rm in}/\omega_{\rm k}$ is increased. When $\nu_{\rm in}/\omega_{\rm k} \ll 1$ (see Fig. 1a for $\nu_{\rm in}/\omega_{\rm k} = 0.1$) we obtain two unstable branches. Compressibility causes the solutions to be closer to the stability threshold, i.e., their growth rates are smaller than in the incompressible case. However, compressibility does not result in the complete stabilisation of all solutions. In particular the solutions with large k_y/k_z cannot be completely stabilised by compressibility. When $\nu_{\rm in}$ and $\omega_{\rm k}$ are of the same order (see Fig. 1b for $\nu_{\rm in}/\omega_{\rm k} = 1$) two unstable branches are present when $k_y/k_z \gg 1$ only (see the solid lines in Fig. 1b corresponding to $k_y/k_z = 10$). However, as k_y/k_z decreases the second branch disappears and only the first branch remains. The behaviour of the first branch is complex because it has two regions of instability. The first one takes place for small velocity shear and is not visible at the scale of Fig. 1b because it has very small growth rates. The second region of instability is the solution visible in Fig. 1b and has much larger growth rates. The threshold shear becomes super-Alfvénic when k_y/k_z decreases. Finally, when $\nu_{\rm in}/\omega_{\rm k} \gg 1$ (see Fig. 1c for $\nu_{\rm in}/\omega_{\rm k} = 10$) only one unstable branch is found. It has two regions of instability as before, but again the first one is not visible at the scale of the Fig. 1c. Eventually, the first unstable region disappears when $\nu_{\rm in}/\omega_{\rm k}$ is increased to a large enough value. The second region of instability remains regardless the value of $\nu_{\rm in}/\omega_{\rm k}$ and has a threshold velocity shear. The threshold shear is super-Alfvénic. Thus, in the limit $\nu_{\rm in}/\omega_{\rm k} \to \infty$ we obtain the behaviour of a plasma where ions, electrons, and neutrals behave as a single-fluid.

Hence, as a summary for high collision frequency and low density contrast between the two media the KHI is suppressed for subalfvenic fluxes, with the neutral fluid being stabilised by the collisions. In other regimes, the KHI might appear, depending on the system parameters. Regarding the particular case of prominence threads, the KHI gives very low growth rates for typical prominence plasma values. The average lifetime of the threads is much shorter than the characteristic time of the instability, and thus this instability does not affect significantly their dynamics and disapearance.

4 Rayleigh-Taylor instability

We study the RTI, with collisions, gravity and compressibility included, but no equilibrium flows. For simplicity, we restrict ourselves to the case $c_A > c_s$. We first study the different collision regimes in Fig. 2. We have computed the growth rate as a function of the dimensionless acceleration of gravity for different values of the collision frequency. Compressibility and collisions work together to lower the linear growth rate, but they are never capable of fully stabilising a configuration that was unstable without these effects. The stability thresholds are not modified and the mode related to the neutral fluid is always unstable. Even the marginal stability criterion for the second mode (related to the ion-electron fluid) is the same that in the uncoupled case. The instability saturates for a large density contrast as in the classical case (Fig. 2b), so increasing the density ratio does not affect too much the results discussed before.

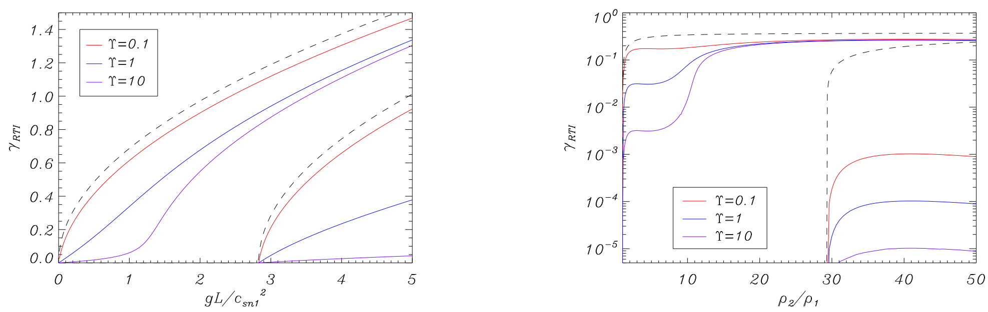

Figure 2: Dimensionless growth rate, $\gamma_{\rm RTI}$ for different values of the dimensionless collision frequency $\Upsilon = \nu_{\rm in} L/c_{\rm sn1}^2$. The dashed lines correspond to the classical collisionless limit ($\Upsilon = 0$). The values $k_x L = 1.0$, $k_y = 1.0$, $\beta = 0.5$ and $\eta_{n1} = \eta_{n2} = 0.5$ have been used in both panels, with $\rho_{2i}/\rho_{1i} = 2$ in the left panel and $gL/c_{\rm sn1}^2 = 0.1$ in the right panel.

In order to perform an application to prominences, we need to assume some values of the parameters of the model. We choose a prominence threads, the fine-structure building blocks of the prominence. In accordance with [7] we choose typical values for the parameters of the model. The linear growth rate is much lower than the prediction of the incompressible collisionless case (by on order of magnitude approximately). Collisions can never stabilise the configuration, but such a low linear growth rate indicates that collisions are efficient enough to significantly reduce the growth rate $\tau_{\rm RTI} \approx 30$ min, which is of the same order as the lifetime of the prominence threads. The classical prediction of this time scale is $\tau_{\rm RTI} \approx 1$ min, much shorter than the reported lifetime of threads [7].

5 Conclusions

Partial ionisation effects are important in certain situations in the solar atmosphere, as prominences, for example. The set of equations for a two-fluid model of neutrals and ions-electrons are discussed. Regarding the instabilities, in general including the neutrals does not modify generally the stability threshold, but tends to lower the linear growth rate. However, each parameter set must be analysed in detail.

For the KHI, for high collision frequency and low density contrast between the two media the KHI is suppressed for subalfvenic fluxes, with the neutral fluid being stabilised by the collisions, but in other cases, the KHI might appear, For prominence threads the characteristic times are higher than the lifetimes, so the instability is not affecting their evolution, despite being present.

For the RTI, we also see that the stability criteria are not modified, but the linear growth rate is lowered. Our results show that the linear time scale of the instability is much shorter in a partially ionised plasma (such as prominences). Thus considering collisions and partial ionisation is a key ingredient that should be added when studying the instability in complex configurations. In fact, the characteristic time scale of the RTI is comparable to the average lifetime of the threads, so the instability plays a relevant role in their evolution and eventual disappearance.

We need to consider further extensions to this work. Fully non-linear analysis is necessary to model the evolution of the system in later stages, and it is currently being carried out. On the other hand, we assumed infinite uniform media in both sides of the discontinuity; this is clearly an approximation, since stratification is not included in our analysis. There are other effects coming from partial ionisation that were not included either, such as electron collisions, non-adiabatic effects, ionisation and recombination processes, finite Larmor radius... The effects on the instabilities beyond the simplified two-fluid model used here are left open for further discussion.

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